Technical Analysis, Liquidity Provision, and Return Predictability

April 29, 2011

Abstract

We develop a strategic trading model to study the liquidity provision role of technical analysis. The equilibrium prices are semi-strong efficient, returns are negatively correlated. Technical traders use past prices to forecast the liquidity premium, which is not related to the fundamental information, and thus provide implicit liquidity to the market in additional to risk-averse liquidity providers. Interestingly, informed traders and technical traders have similar trading patterns: They employ contrarian strategies, their orders have positive price impacts, and forecast short-horizon futures returns. If we interpret the technical traders as individual traders, then our model explain the documented puzzling empirical results regarding individual investors, who are contrarians and whose order are positively related to future returns in the short term. We also show technical trading reduces price predictability and enhances market efficiency by improving the price discovery process, increasing price informativeness, and reducing price impact and price volatility.

Keywords: Technical Analysis, Liquidity Provision, Inventory Effect, Contrarian Strategy, Return Predictability

JEL classification: G11, G12, G14

1 Introduction

Technical analysis is a security analysis discipline to forecast the direction of future prices using primarily past prices and volumes information. Academics do not reach consensus whether technical analysis is useful to investors.[1] However, technical analysis is very popular in the investment industry and considered very useful. Many investment reports and advisory services of brokerage firms are based on technical analysis. Many hedge funds, investment bank, and proprietary trading desks employ some technical trading strategies. Given price and volume data are easily and cheaply available, technical analysis is also very popular among individual investors, who are usually limited by resources and capabilities to conduct fundamental research.

Though technical analysis plays an important role in investment, theoretical analysis of it has been limited. Previous theoretical models usually focus on the role of technical analysis to figure out the information of fundamental value. Instead, this paper takes a different angle and develops a Kyle-type equilibrium model to explore the relationship between technical analysis and liquidity provision.[2] More specifically, we address the following questions: What is the relationship between technical trading and liquidity provision in the short term? What is the effects of technical trading on price discovery process and market quality, such as price informativeness, price variability, price impact. What is its effect on return predictability? What is the trading pattern of technical traders who have no private information? These questions are not widely addressed in existing theoretical studies.[3]

We consider a three-period model. The stock payoff is realized at date 3. Trading game takes place at date 1 and date 2. There are four types of traders: a risk-neutral informed trader, noise traders who trade for exogenous motives, a risk-neutral technical

[1] See detailed discussion in the literature review.

[2] A less contentious empirical finding shows that there exists a positive relationship between technical analysis and liqudity provision. More specifically, Kavajecz and Odders-White (2004) demonstrate that some technical trading rules such as support and resistance levels coincide with peaks in depth on the limit order book and moving average forecasts reveal information about the relative position of depth on the book.

[3] As the subsequent literature review shows, they mainly focus on the information extraction role of technical analysis, but its implications on trading, asset pricing and welfare have not been sufficiently examined. Limited in number, most of these studies are developed in the competitive rational expectation paradigm and rules out the strategic interplay among investors, which we believe is an important feature of technical analysis. One exception is Brunnermeier (2005) who shows that technical analysis can play a role in the strategic rational expectation models under the SEC's Regulation Fair Disclosure. The early-informed traders will employ technical analysis after the public announcement to determine the extent to which the news is already impounded into price. Consequently, their trading behavior is consistent with the street wisdom "buy on (positive) rumors and sell on news".

trader who trade based on historical prices and volumes, and risk-averse (CARA) liquidity providers, who know the equilibrium price and maximize their expected utility at date 3 and their order clear markets. The prices are functions of available order flow and public information. After two-period trading the game ends and all traders receive payments according to the stock payoff and their security holdings. The informed trader knows the stock payoff and the rest only knows its distribution. The informed trader, the technical trader, as well as noise traders submit market orders to competitive risk averse liquidity providers. The informed trader and the technical trader both trade strategically by taking into the impacts of their trades on equilibrium prices. It is worth noting that risk aversion is a tractable way to model the constraints faced by liquidity providers so that they cannot provide costless immediacy services. For example, it may capture risk neutral immediacy subject to funding or VaR constraints.[4] Hence, the liquidity providers are "effective risk averse".

We show that the price comprises two components because of risk aversion, the fair value, which is the fundamental value conditional on liquidity providers' information set, and the liquidity premium to compensate the liquidity providers for holding undesired risky positions. Risk aversion causes the equilibrium price to deviate from the fair value. Since this deviation is not driven by information regarding the fundamental value, a correction is expected in the next period. Hence, returns are negatively autocorrelated. In comparison, when liquidity providers are risk-neutral (face no constraints), the liquidity providers earn zero expected profits because of competition; the price is equal to the fair value, and prices follow a martingale process.[5] The returns are thus independent of each others. Because technical trader provider extra liquidity to the market, the liquidity premium demanded by the liquidity provider is partially reduced, the magnitude of the negative return autocorrelations decline. Hence, the existence of technical trader reduces price predictability and enhances market efficiency.

In our model, the technical trader trades as a contrarian, and his trade positively forecasts future returns. Intuitively, the technical trader knows that by providing extra liquidity to the market, he can earn part of the liquidity premium demanded by the liquidity providers. He thus chooses to trade in the opposite direction of previous price change in period 1. Because he and the informed trader trade taking into account the price impact,

[4]See, e.g., Danielson, Shin, Zigrand (2010) and Adrian, Etula, Shin (2010).

[5]The risk averse liquidity providers are prevalent in the competitive rational expectation paradigm pioneered by Grossman and Stiglitz (1980) and Hellwig (1980) but are relatively rare in the strategic rational expectation model initiated by Kyle (1985). Subrahmanyam (1991) first introduces risk averse market makers into Kyle (1985). The effect of liquidity providers' risk aversion on return dynamics such as momentum, reversal and post-earning announcement drifts is studied in Guo and Kyle (2010).

the price reverses to the fair value only partially. Hence, his position is positively correlated with the next period return in period 3. Because individual investors on average are less informative in comparison with institutional investors, they are more likely to adopt technical analysis to trade. If we interpret the technical trader in our model as Individual investors, these predictions are consistent with numerous empirical findings that individuals tend to be negative feedback traders or contrarians.[6] Our results also explain why the net flows of small investors positively predict future short-horizon returns as documented by Jackson (2003), Kaniel, Saar, and Titman (2008) , and Barber, Odean and Zhu (2009), among others. Since the individual investors lack information advantage, it is usually hard to understand why the trade of individual investors has short term forecasting ability. Our model shows that the forecasting power may come from implicit liquidity provision instead of information advantage. Our model further shows that technical trading is more profitable when the fundamental risk is larger, when the noise trader risk is larger, and (or) when the liquidity providers become more risk averse. Intuitively, an increase in the fundamental risk, noise trader risk, and (or) risk aversion coefficient will cause the prices to be more predictable (measured by return autocorrelation). Kaniel et al. (2008) finds that individual investors' order has more forecasting power for small stock. Small stocks usually involve larger fundamental risk and less liquidity. Hence, all the above results shows that the observed trading patterns of the technical traders is consistent with our story of liquidity provision. It is worth noting that our model shows that liquidity provision can be due to market orders. Remarkably, the technical trader and the informed trader in our model display similar trading patterns: they employ contrarian trading strategy in the second period, their trades have positive price impact on the price, and their trades positively forecasts futures returns. However, as discussed before, they trade for different purposes: the informed trader trades because of his informational advantage whereas the technical trader trades for the purpose of liquidity provision. The implications for empirical studies is that it is not enough to study only the lead-lag or contemporaneous relationships between trades and return to determine whether the trade is informational.

Furthermore, our model also shows that technical trading enhances price discovery process. Compared with the case of no technical trading, price becomes more informative, price volatility declines, and price impacts reduce. Our model demonstrates technical trading in the short run mainly provide implicit liquidity to the markets.

The remainder of the paper is organized as follows. Section 2 reviews several strands

[6] See, Choe, Kho, and Stulz (1999) , Grinblatt and Keloharju (2000, 2001) , Jackson (2003), Richards (2005), Goetzmann and Massa (2002) , and Griffin, Harris, and Topaloglu (2003), among others.

of relevant literature on technical analysis and liquidity provision. Section 3 develops a three-period strategic trading model in which an informed trader and an uninformed traders employing technical analysis trade against risk averse liquidity providers. For the purpose of comparison, we also derive the equilibrium in the absence of uninformed traders. Section 4 studies the impact of technical analysis on trading strategies, liquidity provision, asset pricing patterns, and welfare implications. Section 5 studies several extensions of the benchmark model. Section 6 concludes. All the proofs are delegated to the Appendix.

2 Literature Review

Technical analysis of price and volume patterns has long been regarded as pointless when the efficient market hypothesis is deemed indisputable in the finance research. In fact, this view is strengthened by early study such as Fama and Blume (1960) who show that common filter rules could not outperform the simple buy-and-hold strategy after transaction costs. Economists' enthusiasm in searching for profitable technical analysis is re-ignited when amounted evidence against market efficiency has emerged since 1980s. Influential studies by Brock, Lakonishok, and LeBaron (1992), Lo, Mamaysky, and Wang (2000), LeBron (1999), and Neely (2002), among others, find positive value of some widely employed technical trading rules in stock and foreign exchange markets. However, these studies attract followers as well as critics. Economic and statistical issues such as risk adjustment, data-snooping bias, survivorship bias have been raised and examined (Sullivan,Timmermann, and White,1999; Allen and Karjalainen, 1999), but the controversy seems far away from being settled. According to Park and Irwin (2007), among a total of 95 modern studies, 56 studies find positive results regarding technical trading strategies, 20 studies obtain negative results, and 19 studies indicate mixed results.

The far-reaching popularity and wide employment of technical analysis among practitioners does not receive its theoretical justification until Treynor and Furguson (1985) who defend that past price patterns help a trader to determine whether her information is unique to herself or is common to everyone. In the competitive rational expectation models, Grundy and McNichols (1989), and Brown and Jennings (1989) show that technical analysis of prices has positive value to traders because a single price does not reveal the risky asset's fundamental information but a sequence of prices does under certain conditions. By introducing higher-order uncertainty on quality of asymmetric information, Blume, Easley and O'Hara (1994) show that in addition to learning from price patterns, technical analysis of volumes can improve trader's learning of fundamentals. Traders who use information contained in both will do better than traders who do not. These theoretical rationales for technical analysis are nicely reviewed by Brunnermeier (2001). Nonetheless, these theoretical support does not mean that specific techniques based on recognizing "head-and-shoulers" patterns or "support and resistance levels" and "moving average" can be grounded on the extant models. Zhu and Zhou (2009) attempt to fill the gap by studying the use of moving average, the arguably most popular trading rules, in a standard asset allocation problem. They provide conditions such that fixed allocation rules in conjunction with technical analysis can outperform parameter-dependent optimal learning rule. However, Zhu and Zhou (2009) depart from the prior work on technical analysis in that the asset return process is exogenously given rather than endogenously generated in their model.

We next review the liquidity provision role of market makers and individual traders. Kaniel, Saar and Titman (2008) suggest that their findings that uninformed individual investors employ contrarian and their trade positively forecast the future returns are consistent with the theoretical predications proposed by Grossman and Miller (1988), and Campbell, Grossman, and Wang (1993), that risk-averse individuals act as contrarians to provide liquidity to meet institutional demand for immediacy, and the price concessions provided by the latter naturally lead to subsequent return reversals. The same idea is shared by Hendershott, Li, Menkveld, and Seasholes (2010).

While we largely agree with this wisdom, a number of questions arise in our mind. First, we wonder what drives the individuals to replace the role of market makers who are assigned to take the liquidity provision job. Actually in Grossman and Miller (1998), it is market makers that provide liquidity to immediacy needed individuals and charge for bearing price risk, "although in practice, of course, individuals and firms can play either role at different times." (p.622). If individuals are equally being the demanders and suppliers of immediacy, their trades should not exhibit clear styles and should not be able to forecast future returns. Second, we wonder if the explanation is robust if the lack of immediacy as a result of trading asynchronization is less of a concern in today's markets that are characterized by high trading volume and market liquidity. Last but not the least, we wonder if the risk aversion of individual investors is a pre-condition for the results to hold in the existing theories.

To address these concerns in our strategic trading model we purposefully introduce the market makers, who could be any non-individual entity that provides liquidity, to relieve such responsibility from individual investors. Hendershott et al. (2010) also depart from existing models and demonstrate the liquidity provision role of market makers directly but their objective is different from ours. Moreover, the informed and uninformed investors in their model are risk averse. We consider the market order so that the effect of trading asynchronization is minimized. In general, this type of order will be executed immediately and investors do not have to worry about the delay.

Despite the difference, our analysis is complementary to Grossman and Miller (1988), Campbell, Grossman, and Wang (1993), Hendershott et al. (2010). In particular, we show that the risk aversion of uninformed investors is not a necessary condition for the liquidity provision argument to hold.

Last but not the least, our model can be placed in the literature initiated by Admati and Pfleiderer (1988) which studies the strategic trading of uninformed traders in choosing either the composition or the timing of their trades. We contribute to this literature by showing the rich trading and asset pricing patterns when the informed, uninformed trade strategically against a risk averse liquidity providers.

3 The Model

We develop a three-period trading model in the strategic rational expectation paradigm pioneered by Kyle (1985). In this benchmark economy one risk-free bond and one risky stock are available for trading. We assume that the interest rate for the bond is zero and that the price of the bond is always one for convenience. The stock payoff D is normally distributed with mean $\bar{D}$ and variance σ_D^2. Before trading starts the price of the stock is given by $P_0 = \bar{D}$. The stock payoff is realized at date 3. Trading game takes place at date 1 and date 2 with four types of traders: a risk-neutral informed trader, noise traders who trade for exogenous motives, a risk-neutral technical trader who trades based on historical prices patterns, and a continuum of risk-averse competitive liquidity providers who observe the total order flows, choose trade and price to maximize expected profits and clear the market. After two-period trading the game ends and all traders receive payments according to the stock payoff and their security holdings. The informed trader knows the true value of stock payoff and the rest only knows its distribution. We stress that the model is a short term model because no additional information arrives in the market. The assumption on no dynamic information is used to capture the trading and asset pricing patterns in short horizon.

As in Kyle (1985), trading in each period $t \in \{1, 2\}$ occurs in two steps. In the first step, the informed trader, the technical trader, and noise traders submit their market orders. In the second step, the liquidity provider observes the net order flows, sets the prices and clears the market. Except for the noise traders, other traders choose trading strategies to maximize their expected profits respectively. The informed trader initiates trading and chooses his optimal trading strategies X_t at date t. The technical trader does not trade at date 1 but picks his optimal trading quantity Z at date 2 conditional on the historical prices information P_0 and P_1. The noise traders submit their total orders U_t at date t where U_t are normally distributed with mean zero and variance σ_U^2. In addition, U_1 and U_2 are independent from each other and from the stock payoff D. Without loss of generality the total mass of liquidity providers is normalized to unity. We depart from Kyle (1985) by assuming that the representative liquidity provider has a CARA utility function $-\exp\left(-\gamma W_3^Y\right)$, where γ denotes the risk-aversion coefficient, and W_3^Y denotes the liquidity provider's accumulated wealth at date 3. After observing the the total order flows ω_t in period t submitted by other traders, where

$$\omega_1 = X_1 + U_1$$
$$\omega_2 = X_2 + Z + U_2$$

the representative liquidity provider chooses her optimal positions Y_t and sets the competitive prices P_t not only to maximize her expected utility but also to clear the market. The initial endowments of all traders are normalized to be zero to simplify the analysis without affecting any propositions. As will be evident in the subsequent analysis, the risk aversion assumption of the liquidity provider is essential for our main results while the risk neutrality assumption of the informed and technical traders are just for convenience.

Next, we specify the equilibrium stock price, the market clearing condition and the equilibrium trading strategies by the liquidity provider, the informed trader and the technical trader respectively in each period. Following the convention we restrict our attention to a linear equilibrium, which is decomposed into three parts. First, in each period the stock price is a linear function of liquidity providers' state variables. She sets equilibrium price from the conjectured trading strategies of the informed and technical traders and her updated beliefs about the stock payoff. She maximizes her expected utility by choosing her position in stock which clears the market. Second, given the pricing rule, the liquidity provider' updated beliefs, and the technical trader's trading strategy in each period, the informed investor figures out his investment opportunities and chooses his optimal trading strategy. His order flow is a linear function of his state variables. Third, given the pricing rule, the liquidity provider's updated beliefs, and the informed trader's trading strategy in each period, the technical trader chooses his optimal trading strategy, which is a linear function of past prices.

To solve the linear equilibrium, we begin by postulating that the equilibrium stock prices in each period are given by

$$P_1 = \lambda_{10} + \lambda_{11}\omega_1, \tag{3.1}$$

$$P_2 = \lambda_{20} + \lambda_{21}\omega_1 + \lambda_{22}\omega_2. \tag{3.2}$$

The coefficients $\lambda' s$ are constants to be determined in the equilibrium and λ_{11}, λ_{21} and λ_{22} are liquidity parameters. Because the liquidity provider sets prices based on her observed order flows of other traders, two implications follow naturally. First, P_t depends on the order flows up to period t therefore price in period 2 is a function of ω_1 and ω_2. Second, the liquidity provider's optimal holdings Y_t in the stock do not appear in the price functions.

To clear the market clears, the sum of the positions of the informed trader, the technical trader, noise traders, and the liquidity provider must be equal to zero in each period:

$$X_1 + U_1 + Y_1 = 0, \tag{3.3}$$

$$X_2 + Z + U_2 + Y_2 = 0. \tag{3.4}$$

Using the pricing functions and the market clearing conditions, we are able to rigorously solve the dynamic maximization problems of the informed trader, the technical trader, and the liquidity provider to determine their optimal trading strategies as well as the coefficients in the pricing functions.

We conjecture that the linear trading strategies of the informed trader in each period are given by

$$X_1 = \beta_{10} + \beta_{11}D, \tag{3.5}$$

$$X_2 = \beta_{20} + \beta_{21}D + \beta_{22}\omega_1 + \beta_{23}Z. \tag{3.6}$$

The coefficients $\beta' s$ are constants to be determined in the equilibrium and β_{11}, β_{21} and β_{22} measure informed trader's trading intensity. It is worth noting that the informed trader trades only on private information in period 1, but in period 2 he trades on private information as well as date-2 public information ω_1, which is informationally equivalent to price P_1. More importantly, because the technical trader's order Z depends on P_0 and P_1 which are publicly observable, the informed trader could infer this quantity in equilibrium and will therefore take into account its impact on the equilibrium price. Consequently, the informed trader's trading strategy in period 2 also depend on Z.

Since observing P_1 is equivalent to observing ω_1, we conjecture that the linear trading strategy of the technical trader in period 2 is given by

$$Z = h_0 + h_1 \omega_1. \tag{3.7}$$

The coefficients $h's$ are constants to be determined in the equilibrium and h_1 measures technical trader's trading intensity.[7] The uninformed technical trader does not trade in period 1 to avoid losing money to the informed.

Taking above analysis together, we see that while the technical trader is a pure technician, the informed trader is a partial fundamentalist and a partial technician. Both exploit the historical prices and order flows patterns, which are informationally equivalence in the benchmark economy, in the trading strategies. We will see in the subsequent equilibrium analysis that such a technical analysis has significant impact on the liquidity provision, trading strategies, price impacts, return predictability, etc.

We also assume that based on the liquidity provider's information set, the expected values of stock payoff in each period are given by

$$E[D|\mathcal{F}_1] = \tau_{10} + \tau_{11}\omega_1, \tag{3.8}$$

$$E[D|\mathcal{F}_2] = \tau_{20} + \tau_{21}\omega_1 + \tau_{22}\omega_2, \tag{3.9}$$

where $\mathcal{F}_t$ denotes the information set of the liquidity provider in period t, thus $\mathcal{F}_1 = \{\omega_1\}$ and $\mathcal{F}_2 = \{\omega_1, \omega_2\}$. The coefficients $\tau's$ are constants to be determined in the equilibrium.

In the ensuing analysis, we solve a linear equilibrium in which above conjectures are confirmed to be correct. Note that it does not matter which trader's profit maximization problem is to be solved first.

3.1 The Informed Trader's Maximization Problems

The informed trader's optimization problems is derived using backward induction. We first solve the maximization problem in period 2. Taking the optimal solution for this period as given, we then solve the maximization problem in period 1. Denote the informed trader's final wealth at date 3 by W_3^X. Recall that all traders' initial wealth equals zero,

$$W_3^X = X_1(D - P_1) + X_2(D - P_2). \tag{3.10}$$

[7] To break this equivalence we need to introduce additional uncertainty so that both price and volume provide useful information in the technical analysis. See Wang (1994) and Blume, Easley and O'Hara (1994).

The informed trader's maximization problem in period 2 is

$$\max_{X_2} E\left[X_2(D-P_2)|\mathcal{F}_2^I\right],$$

where $\mathcal{F}_t^I$ denotes the information set of the informed investor at date $t \in \{1,2\}$. Plugging in the conjectured price function (3.2) we obtain the first-order condition (FOC) with respect to X_2

$$(D - E[\lambda_{20} + \lambda_{21}\omega_1 + \lambda_{22}\omega_2|\mathcal{F}_2^I]) - \lambda_{22}X_2 = 0. \tag{3.11}$$

Rearrangement gives

$$X_2 = \frac{D - \lambda_{20} - \lambda_{21}\omega_1 - \lambda_{22}Z}{2\lambda_{22}}. \tag{3.12}$$

Clearly, the second-order condition (SOC) is negative when $\lambda_{22} > 0$. Hence, (3.12) yields the optimal net holding in period 2.

Plugging (3.11) in (3.10), the informed trader chooses X_1 to maximize his final wealth:

$$\max_{X_1}\left[X_1(D - E[P_1|\mathcal{F}_1^I]) + \lambda_{22}E[X_2^2|\mathcal{F}_1^I]\right].$$

To solve the problem, the insider substitutes in (3.1), (3.6). The FOC with respect to X_1 yields

$$X_1 = \frac{[2\lambda_{22}(\beta_{20} + \beta_{23}h_0)(\beta_{22} + \beta_{23}h_1) - \lambda_{10}] + [1 + 2\lambda_{22}\beta_{21}(\beta_{22} + \beta_{23}h_1)]D}{2\lambda_{11} - 2\lambda_{22}(\beta_{22} + \beta_{23}h_1)^2} \tag{3.13}$$

This is the optimal trade chosen by the informed trader in period 1 if the SOC is negative, i.e.,

$$\lambda_{11} - \lambda_{22}(\beta_{22} + \beta_{23}h_1)^2 < 0. \tag{3.14}$$

3.2 The Technical Trader's Maximization Problem

Denote the technical trader's final wealth by $W_3^Z = Z(D - P_2)$. Because he chooses his optimal trade based on P_0 and P_1, his information set at date 2 is the same as the information set $\mathcal{F}_1$ of the liquidity provider in period 1. Using (3.2) and (3.9), his maximization problem in period 2 is given by

$$\max_Z Z\left[(\tau_{10} + \tau_{11}\omega_1) - (\lambda_{20} + \lambda_{21}\omega_1 + \lambda_{22}E[X_2|\mathcal{F}_1] + \lambda_{22}Z)\right].$$

Because of (3.6), the FOC with respect to Z yields

$$(\tau_{10} + \tau_{11}\omega_1) - (\lambda_{20} + \lambda_{21}\omega_1 + \lambda_{22}E[X_2|\mathcal{F}_1] + \lambda_{22}Z) - \lambda_{22}(1 + \beta_{23})Z = 0.$$

Note that the technical trader will take into account the fact that the informed trader's trading in period 2 depends on Z. Substituting in (3.6) and (3.8) we obtain

$$Z = \frac{[\tau_{10} - \lambda_{20} - \lambda_{22}(\beta_{20} + \beta_{21}\tau_{10})] + [\tau_{11} - \lambda_{21} - \lambda_{22}(\beta_{21}\tau_{11} + \beta_{22})]\omega_1}{2\lambda_{22}(1 + \beta_{23})}. \tag{3.15}$$

We show in the Appendix that $1 + \beta_{23} > 0$, so the SOC is negative when $\lambda_{22} > 0$. Hence, (3.15) gives the optimal solution.

3.3 The Liquidity Provider's Maximization and Beliefs Updating Problems

The liquidity provider's final wealth W_3^Y satisfies

$$W_3^Y = Y_1(D - P_1) + Y_2(D - P_2). \tag{3.16}$$

The maximization problem of the liquidity provider with CARA utility is equivalent to

$$\max_{Y_2} \left[E\left[W_3^Y | \mathcal{F}_2 \right] - \frac{1}{2}\gamma Var\left[W_3^Y | \mathcal{F}_2 \right] \right],$$

Plugging in (3.16) yields

$$\max_{Y_2} \left[(E\left[D|\mathcal{F}_2\right] - P_2)Y_2 + (E\left[D|\mathcal{F}_2\right] - P_1)Y_1 - \frac{1}{2}\gamma Var(D|\mathcal{F}_2)(Y_1 + Y_2)^2 \right].$$

The first-order condition (FOC) with respect to Y_2 yields the optimal net holdings in period 2

$$Y_2 = \frac{E[D|\mathcal{F}_2] - P_2}{\gamma Var\left[D|\mathcal{F}_2\right]} - Y_1. \tag{3.17}$$

Note that the first term is the familiar demand function for the stock, which increases with the expected excess return for investing in the stock, and decreases with both the risk aversion of the liquidity provider and the risk of the stock payoff. Because the liquidity provider is risk averse, the second term shows that her demand for the risky stock decreases with her cumulative holdings in the stock. Put it another way, $Y_1 + Y_2$ is the optimal total holdings of the liquidity provider in period 2.

In period 1, plugging (3.17) into the liquidity provider's wealth (3.16) gives

$$W_3^Y = Y_1(P_2 - P_1) + \frac{(E[D|\mathcal{F}_2] - P_2)(D - P_2)}{\gamma Var\left[D|\mathcal{F}_2\right]}.$$

By the law of iterated expectation, the liquidity provider's expected utility, conditional her information set, is given by

$$\max_{Y_1} Y_1 \left[E[P_2|\mathcal{F}_1] - P_1\right] + E\left[\left.\frac{(E[D|\mathcal{F}_2] - P_2)^2}{\gamma Var\left[D|\mathcal{F}_2\right]}\right|\mathcal{F}_1\right] - \frac{\gamma}{2} Var\left[W_3^Y|\mathcal{F}_1\right].$$

The FOC with respect to Y_1 yields

$$Y_1 = \frac{E[P_2|\mathcal{F}_1] - P_1 - a\gamma}{\gamma Var[P_2|\mathcal{F}_1]}, \tag{3.18}$$

where

$$a = Cov\left[P_2 - P_1, \left.\frac{(E[D|\mathcal{F}_2] - P_2)(D - P_2)}{\gamma Var\left[D|\mathcal{F}_2\right]}\right|\mathcal{F}_1\right]$$

The following lemma is very helpful for deriving the liquidity provider's optimal trading in period 1.

Lemma 1 *We have $a = 0$.*

Proof. All proofs are provided in the Appendix. ■

Therefore, the optimal trading strategy of the liquidity provider given in (3.18) is reduced to

$$Y_1 = \frac{E[P_2|\mathcal{F}_1] - P_1}{\gamma Var[P_2|\mathcal{F}_1]}. \tag{3.19}$$

The optimal solutions (3.17) and (3.19) show that the liquidity provider's maximization problem is static. Importantly, such a behavior is endogenously optimal, which is different from the myopic behavior studied by Brown and Jennings (1989), and Blume, Easley and O'Hara (1994).

We next turn to the beliefs updating problem of the liquidity provider. The trading strategies of the informed trader and the technical trader are given by (3.5), (3.6) and (3.7) respectively, the liquidity provider directly applies the projection theorem for normally distributed random variables to derive the optimal updating rules in (3.8) and (3.9). In period 1 we have

$$\tau_{11} = \frac{\beta_{11}\sigma_D^2}{\beta_{11}^2\sigma_D^2 + \sigma_U^2}, \tag{3.20}$$

$$\tau_{10} = -\tau_{11}\beta_{10}. \tag{3.21}$$

In period 2, τ_{21}, τ_{22} and τ_{20} respectively satisfy

$$\begin{pmatrix} \tau_{21} \\ \tau_{22} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix}^{-1} \begin{pmatrix} \beta_{11}\sigma_D^2 \\ \left[\beta_{21} + (\beta_{22} + (1+\beta_{23})h_1)\beta_{11}\right]\sigma_D^2 \end{pmatrix} \tag{3.22}$$

where

$$c_0 = \sigma^2_{\omega_1} = \beta^2_{11}\sigma^2_D + \sigma^2_U, \tag{3.23}$$

$$c_1 = \sigma_{\omega_1\omega_2} = \beta_{11}\left[\beta_{21} + (\beta_{22} + (1+\beta_{23})h_1)\beta_{11}\right]\sigma^2_D + \left[\beta_{22} + (1+\beta_{23})h_1\right]\sigma^2_U, \tag{3.24}$$

$$c_2 = \sigma^2_{\omega_2} = [\beta_{21} + (\beta_{22} + (1+\beta_{23})h_1)\beta_{11}]^2\sigma^2_D + [1 + (\beta_{22} + (1+\beta_{23})h_1)^2]\sigma^2_U. \tag{3.25}$$

and

$$\tau_{20} = -\tau_{21}\beta_{10} - \tau_{22}[(\beta_{22} + (1+\beta_{23})h_1)\beta_{10} + \beta_{20} + (1+\beta_{23})h_0]. \tag{3.26}$$

3.4 Equilibrium Trades and Prices

We have derived traders' trading strategies and solved the liquidity provider's beliefs updating problem. Note that the equilibrium characterization will be greatly simplified if we normalize the mean of the stock payoff $\bar{D}$ to be zero. This assumption will not affect any of our propositions regarding trading and asset pricing patterns.

Lemma 2 *When the mean of the stock payoff is normalized to zero, the first constants in the conjectured pricing functions (3.1)-(3.2), trading strategies (3.5)-(3.6) and updating rules (3.8)-(3.9) are all equal to zero.*

By equating coefficients of the derived trading strategies and the conjectured strategies, and using the market clearing conditions we obtain and summarize the equilibrium trades and prices in Theorem 1.

Theorem 1 *When an linear equilibrium exists in the benchmark economy, equilibrium trading strategies of an informed trader and a technical trader, and equilibrium pricing strategy of an liquidity provider are given by*

$$X_1 = \beta_{11}D, \qquad X_2 = \beta_{21}D + \beta_{22}(X_1 + U_1) + \beta_{23}Z, \tag{3.27}$$

$$Z = h_1(X_1 + U_1), \tag{3.28}$$

$$P_1 = \lambda_{11}(X_1 + U_1), \qquad P_2 = \lambda_{21}(X_1 + U_1) + \lambda_{22}(X_2 + Z + U_2), \tag{3.29}$$

where

$$\beta_{11} = \frac{8\lambda_{22} - 2(\tau_{11} + \lambda_{21})}{16\lambda_{11}\lambda_{22} - (\tau_{11} + \lambda_{21})^2}, \tag{3.30}$$

$$\beta_{21} = \frac{1}{2\lambda_{22}}, \qquad \beta_{22} = -\frac{\lambda_{21}}{2\lambda_{22}}, \qquad \beta_{23} = -\frac{1}{2}, \tag{3.31}$$

$$h_1 = \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22}}, \tag{3.32}$$

$$\lambda_{11} = \frac{3\tau_{11} + \lambda_{21}}{4} + \gamma Var[P_2|\mathcal{F}_1], \tag{3.33}$$

$$\lambda_{21} = \tau_{21} + \gamma Var[D|\mathcal{F}_2], \qquad \lambda_{22} = \tau_{22} + \gamma Var[D|\mathcal{F}_2], \tag{3.34}$$

$$Var[P_2|\mathcal{F}_1] = \begin{pmatrix} \lambda_{21} & \lambda_{22} \end{pmatrix} \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix} - \frac{(\lambda_{21}c_0 + \lambda_{22}c_1)^2}{c_0}, \tag{3.35}$$

$$Var[D|\mathcal{F}_2] = \sigma_D^2 - \begin{pmatrix} \tau_{21} & \tau_{22} \end{pmatrix} \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \tau_{21} \\ \tau_{22} \end{pmatrix}, \tag{3.36}$$

and τ_{11} satisfies (3.20) and τ_{21} and τ_{22} satisfy (3.22), and c_0, c_1 and c_2 are given in (3.23)-(3.24). The second-order conditions are

$$16\lambda_{11}\lambda_{22} < (\tau_{11} + \lambda_{21})^2 \quad \textit{and} \quad \lambda_{22} > 0.$$

We have derived the general expressions of the endogenous parameters in terms of exogenous parameters. Despite the complexity, we can employ the numerical solutions to obtain the values and study the comparative statics. Because our main goal is to investigate the impact of technical analysis on trading and asset pricing patterns such as trading strategy, market liquidity, price informativeness, and return predictability, and we are particularly keen on understanding the magnitude difference of these patterns in response to same exogenous parameters. In order to illustrate the effects of technical trader more clearly, we introduce two special cases in which the technical trader does not trade. In the first case, only an informed trader and noise traders trade against a risk averse liquidity provider. In the second, the risk averse liquidity provider is replaced by a risk neutral one. We summarize the equilibrium conditions of these two special cases in Theorem 2 and Theorem 3 respectively.

Theorem 2 *When an linear equilibrium exists in the benchmark economy without technical trader, equilibrium conditions are given by*

$$X_1 = \beta_{11} D, \qquad X_2 = \beta_{21} D + \beta_{22}(X_1 + U_1),$$
$$P_1 = \lambda_{11}(X_1 + U_1), \qquad P_2 = \lambda_{21}(X_1 + U_1) + \lambda_{22}(X_2 + U_2),$$

where

$$\beta_{11} = \frac{2\lambda_{22} - \lambda_{21}}{2\lambda_{11}\lambda_{22} - \lambda_{21}^2},$$
$$\lambda_{11} = \frac{\tau_{11} + \lambda_{21}}{2} + \gamma Var\left[P_2|\mathcal{F}_1\right].$$

and β_{21}, β_{22}, λ_{21}, λ_{22}, *and* τ_{11}, τ_{21}, τ_{22} *have the same expressions as in the Theorem 1 except that* c_0, c_1 *and* c_2 *are replaced by*

$$d_0 = \beta_{11}^2\sigma_D^2 + \sigma_U^2$$
$$d_1 = \beta_{11}\left(\beta_{21} + \beta_{22}\beta_{11}\right)\sigma_D^2 + \beta_{22}\sigma_U^2$$
$$d_2 = \left(\beta_{21} + \beta_{22}\beta_{11}\right)\sigma_D^2 + \left(1 + \beta_{22}^2\right)\sigma_U^2$$

respectively. The second-order conditions are

$$4\lambda_{11}\lambda_{22} < \lambda_{21}^2 \quad \textit{and} \quad \lambda_{22} > 0.$$

When the liquidity provider is risk neutral, we can immediately conclude that technical trader will not trade. In this case, we obtain $E[P_2|\mathcal{F}_1] = E[D|\mathcal{F}_1]$. Intuitively, the price has the martingale property. Hence, $E[Z(D - P_2)|\mathcal{F}_1] = ZE[(D - P_2)|\mathcal{F}_1] = 0$. In our setup, the existence of risk neutral liquidity providers would have already exploited any expected profit opportunity based on information inferred from past prices. Therefore, these opportunities cannot arise in equilibrium and thus technical analysis has no value.

Theorem 3 *Technical trader will not trade when the liquidity provider is risk neutral. The linear equilibrium trading strategies of an informed trader and the linear equilibrium pricing strategy of the liquidity provider are given by*

$$X_1 = \beta_{11}D, \qquad X_2 = \beta_{21}D + \beta_{22}\left(X_1 + U_1\right),$$
$$P_1 = \lambda_{11}\left(X_1 + U_1\right), \qquad P_2 = \lambda_{21}\left(X_1 + U_1\right) + \lambda_{22}\left(X_2 + U_2\right),$$

where

$$\beta_{11} = \frac{2L-1}{4L-1}\frac{1}{\lambda_{11}}, \qquad \beta_{21} = \frac{1}{2\lambda_{22}}, \qquad \beta_{22} = -\frac{\lambda_{21}}{2\lambda_{22}},$$
$$\lambda_{11} = \lambda_{21} = \frac{\sqrt{2L\left(2L-1\right)}}{4L-1}\frac{\sigma_D}{\sigma_U}, \qquad \lambda_{22} = \sqrt{\frac{L}{2\left(4L-1\right)}\frac{\sigma_D}{\sigma_U}},$$

where

$$L = \frac{\lambda_{22}}{\lambda_{11}} = \frac{1}{6}\left[1 + 2\sqrt{7}\cos\left(\frac{\pi - \arctan 3\sqrt{3}}{3}\right)\right] \approx 0.901$$

is the unique positive solution to a cubic equation $8L^3 - 4L^2 - 4L + 1 = 0$.

4 Equilibrium Analysis

We carry out the equilibrium analysis in the following sections. We first consider the effect of technical analysis on asset pricing, including price informativeness, price variability, price impact, return autocorrelations. We then examine the trading patterns of the technical trader and the informed trader. We are particularly interested in addressing why technical analysis is so popular among professional as well as amateur investors.

4.1 Price Informativeness, Price Variability, Price Impact and Market Liquidity

In this section, we study the equilibrium properties of the price informativeness, price variability, market liquidity, and price impact. The price informativeness can be measured by

$$PI = 1 - \frac{Var\left[D|\mathcal{F}_2\right]}{\sigma_D^2}, \tag{4.1}$$

namely, the percentage of private information being incorporated into the prices after 2-period trading. Note that $Var\left[D|\mathcal{F}_2\right]$ reflects the remaining variance of the distribution so a lower value correspond to a more efficient or informative price. For simplicity, we measure the stock price variability by

$$PV = \frac{Var\left[D - P_2\right]}{\sigma_D^2}, \tag{4.2}$$

which is the variance of dollar return from date 2 to date 3 normalized by the variance of stock payoff. λ_{11}, λ_{21}, and λ_{22} measure the price impact coefficients and τ_{11}, τ_{21}, and τ_{22} measure the updating coefficient of private information regarding the total order flows and thus the price impact due to asymmetric information rather than inventory effect. Following Kyle (1985), price impact measure is the inverse of market liquidity, which is the order flow necessary to induce the price to rise or fall by one dollar.

We first study the effect of fundamental risk σ_D and keep other exogenous parameters constant. Figure 1 plots these measures against σ_D. For comparison, we include the measures for the case of risk-neutral liquidity providers and the case of risk-averse liquidity providers without the technical trader. An increase in fundamental risk σ_D implies that a larger amount of private information, therefore the liquidity providers require a larger compensation to combat the adverse selection, which in turn results in a larger price impact. Besides this asymmetric information effect, an increase in stock payoff volatility also leads to a larger price impact due to inventory effect since liquidity providers asks for higher compensation for bearing more risk for each undesired stock holding when providing immediacy to other traders. These two effects make price impacts measured by τ_{11}, τ_{21}, τ_{22} and λ_{11}, λ_{21}, λ_{22} increase as shown in panels C through H of Figure 1, therefore the market becomes less liquid. When the uncertainty rises, informed trader foresees the upward price adjustment of liquidity providers, and he reduces his trading intensities β_{11} and β_{21} on his private information, but the patterns of β_{22} and h_1 in panels K and L of Figure 1 show the informed trades more aggressively on the inferred period-1 order flow ω_1, so does the technical trader. Recall that β_{22} and h_1 describes the extent of technical analysis employed by the informed and uninformed respectively.[8] Because period 2 price positively responds to order flows ω_1 and ω_2 as measured by λ_{21} and λ_{22} respectively, the informed and uninformed traders' trading intensity on ω_1 is negatively related to λ_{21} and positively related to λ_{22}. Thus the magnitude of β_{22} and h_1 depends on the relative value of λ_{21} and λ_{22}. Figure 1 show that when the fundamental risk increases, both the informed and uninformed rely more heavily on technical analysis. These considerations lead to a more complicated patterns of price informativeness and price variability, compared to the those in Kyle (1985). As shown in panels A and B, the combined effect of changing trading intensities in response to increasing uncertainty generates a higher price informativeness and a lower price variability. From a different perspective, when price P_2 becomes more informative, it incorporates more private information so price volatility tend to be smaller on the one hand (as most evident in the case of risk neutral liquidity provider), but a larger σ_D also cause P_2 to deviate more from the fundamental value due to the inventory risk effect on the other. Consequently, the change of price variability is mixed. Our numerical analysis shows that in general the former effect dominates the latter for given parameter values. It deserves mentioning that a higher fundamental risk directly contributes to a higher value of PI and a lower value of PV because of the normalization as shown in (4.1) and (4.2).

[8]Note that in equilibrium $\beta_{23} = -1/2$, so the responsiveness of the informed trader to the order submitted by the technical trader is a negative constant, regardless of the exogenous parameters. This follows from a direct strategic substitution effect. As the technical trader becomes more aggressive in trading, the informed trader lower its trading intensity in order to reduce the total order's adverse impact on the price.

Second, we study the effect of noise trader risk σ_U and keep other exogenous parameters constant. Figure 2 plots these measures against σ_U. An increase in σ_U implies a smaller compensation to liquidity providers for adverse selection, and thus a small price impact due to the asymmetric information effect. Both $\tau's$ and $\lambda's$ decrease as a consequence. The strategic response of the informed trader is to trade more aggressively on private information. Panels K and L of Figure 2 show that the informed and the uninformed again trade more intensely on the inferred period 1 order flow ω_1 as dictated by the magnitude of λ_{21} and λ_{22} for given parameters. To understand the effect of σ_U on the price informativeness PI and price variability PV. We first note that in the case of risk neutral liquidity provider, both PI and PV are constant. As revealed by Kyle (1985), a higher σ_U implies more noise trading which in turn provides more opportunity for the informed to disguise his information based trading. The informed trades more aggressively but the impact on PI and PV are unchanged. This reasoning no longer holds when technical trading become a part of the game. The combined effect is a lower PI but a higher PV.

Third, we study the effect of risk aversion coefficient γ and keep other exogenous parameters constant. Figure 3 plots these measures against γ. From previous analysis, we expect the effect of higher γ on market liquidity and traders' trading intensity are similar to the effect of higher σ_D. Panels F through L of Figure 3 confirm this reasoning. But in contrast to a higher σ_D, the patterns of price informativeness PI and price variability PV are quite distinct as we observe a lower PI and a higher PV for given parameter values. We believe this is due to the lack of normalization effect as a higher γ only affects PI and PV, as shown in (4.1) and (4.2) respectively, indirectly. In this case, when price becomes less informative, its volatility tends to be higher. Moreover, a larger γ causes P_2 to deviate more from the fundamental value due to inventory effect, the price variability PV becomes larger.

We stress that the effect of fundamental risk, noise trader risk and liquidity provider's risk aversion on the price informativeness, price variability, price impact, market liquidity and trading intensities are largely consistent across a wide range of parameters. It is also noteworthy that these patterns are largely uniform regardless of the existence of technical trading when the liquidity provider is risk averse, with the only exception of the patterns regarding trading intensity β_{22}, which are determined by the magnitude of λ_{21} and λ_{22} and exhibits a "U"-shape against exogenous parameters. In the presence of technical analysis, quite often the liquidity provider assigns a higher value to λ_{21} because ω_1 is heavily exploited in period-2 trading, therefore β_{22} lies in the declining part of a "U"-shape curve. In contrast, in the absence of technical analysis, the liquidity provider relies more on λ_{22} in the price adjustment, hence β_{22} often lies in the rising part of a "U"-shape curve.

When liquidity provider is risk averse, our numerical analysis also shows that price impacts are smaller, the price is more informative, and the price variability is smaller in the benchmark economy when compared to the case without technical trader. Intuitively, the technical trader provides implicit liquidity to the informed trader and noise traders. Because of the competition from the technical trader, the risk-averse liquidity providers demand smaller compensations. Our results are broadly consistent with the empirical finding shown in Kavajecz and Odders-White (2004).

4.2 Return Autocorrelations

This section studies the time series properties of the returns (price changes). The returns in each periods are defined as: $r_1 = P_1 - P_0$, $r_2 = P_2 - P_1, r_3 = D - P_2$. The predictability of stock returns using historical prices is a necessary condition for the technical trader to trade. To illustrate the impact of technical analysis, we compare the magnitude of return autocorrelations with or without technical trader. The results are summarized in Proposition 1.

Proposition 1 *(1) When the liquidity provider is risk averse: (a) In the presence of technical trader successive price changes satisfy*

$$Corr\left[P_1 - P_0, P_2 - P_1\right] = \frac{\lambda_{11}\left(3\tau_{11} + \lambda_{21} - 4\lambda_{11}\right)c_0}{4\sqrt{Var\left[P_1 - P_0\right]Var\left[P_2 - P_1\right]}},$$
$$Corr\left[P_2 - P_1, D - P_2\right] = \frac{\left(\tau_{21} - \lambda_{21}\right)\left[\left(\lambda_{21} - \lambda_{11}\right)\left(c_0 + c_1\right) + \lambda_{22}\left(c_1 + c_2\right)\right]}{\sqrt{Var\left[P_2 - P_1\right]Var\left[D - P_2\right]}},$$
$$Corr\left[P_1 - P_0, D - P_2\right] = \frac{h_1\lambda_{11}\lambda_{22}c_0}{2\sqrt{Var\left[P_1 - P_0\right]Var\left[D - P_2\right]}},$$

where the parameters are characterized in the Theorem 1 and

$$Var\left[P_1 - P_0\right] = \lambda_{11}^2 c_0, \tag{4.3}$$
$$Var\left[P_2 - P_1\right] = \left(\lambda_{21} - \lambda_{11}\right)^2 c_0 + 2\left(\lambda_{21} - \lambda_{11}\right)\lambda_{22}c_1 + \lambda_{22}^2 c_2, \tag{4.4}$$
$$Var\left[D - P_2\right] = \sigma_D^2 + \begin{pmatrix} \lambda_{21} - 2\tau_{21} & \lambda_{22} - 2\tau_{22} \end{pmatrix}\begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix}\begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix}. \tag{4.5}$$

(b) In the absence of technical trader successive price changes satisfy

$$Corr\left[P_1 - P_0, P_2 - P_1\right] = \frac{\lambda_{11}\left(\tau_{11} + \lambda_{21} - 2\lambda_{11}\right) d_0}{2\sqrt{Var\left[P_1 - P_0\right] Var\left[P_2 - P_1\right]}},$$

$$Corr\left[P_2 - P_1, D - P_2\right] = \frac{\left(\tau_{21} - \lambda_{21}\right)\left[\left(\lambda_{21} - \lambda_{11}\right)\left(d_0 + d_1\right) + \lambda_{22}\left(d_1 + d_2\right)\right]}{\sqrt{Var\left[P_2 - P_1\right] Var\left[D - P_2\right]}},$$

$$Corr\left[P_1 - P_0, D - P_2\right] = \frac{\lambda_{11}\left(\tau_{11} - \lambda_{21}\right) d_0}{2\sqrt{Var\left[P_1 - P_0\right] Var\left[D - P_2\right]}}$$

where the parameters are characterized in the Theorem 2 and

$$Var\left[P_1 - P_0\right] = \lambda_{11}^2 d_0,$$

$$Var\left[P_2 - P_1\right] = \left(\lambda_{21} - \lambda_{11}\right)^2 d_0 + 2\left(\lambda_{21} - \lambda_{11}\right)\lambda_{22} d_1 + \lambda_{22}^2 d_2,$$

$$Var\left[D - P_2\right] = \sigma_D^2 + \begin{pmatrix} \lambda_{21} - 2\tau_{21} & \lambda_{22} - 2\tau_{22} \end{pmatrix} \begin{pmatrix} d_0 & d_1 \\ d_1 & d_2 \end{pmatrix} \begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix}.$$

(2) When the liquidity provider is risk neutral, successive price changes satisfy

$$Corr\left[P_1 - P_0, P_2 - P_1\right] = Corr\left[P_2 - P_1, D - P_2\right] = Corr\left[P_1 - P_0, D - P_2\right] = 0.$$

[Insert Figures 4-6 Here]

Figures 4 through 6 show that stock returns are negatively autocorrelated in this economy when the liquidity providers are risk averse. The negative autocorrelation is a direct results of inventory effect due to the risk aversion of the liquidity provider. Risk aversion causes the equilibrium price to deviate from the fair value which is the conditional expected value of stock payoff based on the liquidity provider's information set. Since this deviation is not driven by information regarding the fundamental value, a correction is expected in the next periods. Hence, returns are negatively autocorrelated. A clearer explanation relies on the discussion of the trading strategies of the technical trader and the informed trader, which will be provided in sections 4.3 and 4.4. Consistent with the economic intuition, the numerical analysis in Figures 4 through 6 shows that an increase in fundamental risk σ_D, noise trader risk σ_U, or liquidity provider's risk aversion γ leads to a larger risk premium required by the liquidity provider to take one share of undesired stock, and thus larger magnitude of return reversal. Hence, returns are more negative return autocorrelations and more predictable.

Empirical studies such as Jegadeesh (1990), Lehmann (1990), and Kaul and Mimalendran (1990) have shown that stock returns tend to reverse at horizons ranging from a week to a month. As mentioned before, in our model the information structure and short date interval are used to capture the short-term trading. The inventory effect is driven by risk aversion of liquidity providers, thus our model is in line with the inventory theory developed by Stoll (1978), Roll (1984), O'Hara and Oldfield (1986), and Grossman and Miller (1988), among others.[9] Subrahmanyam (2005). Our model differs from the previous work by introducing uninformed investors' technical analysis and exploring the effect of technical trading on return predictability. We focus on the predictability of $D - P_2$ by past returns. Figures 4 through 6 show that, compared to the return autocorrelations in absence of technical trading, the magnitude of $Corr\left[P_2 - P_1, D - P_2\right]$ is larger whereas $Corr\left[P_1 - P_0, D - P_2\right]$ is smaller when technical trader is involved. In fact, these patterns are observed for a wide range of parameters and they are the consequence of the negative correlation between $P_1 - P_0$ and $P_2 - P_1$. To further study the effect of technical trading on the predictability of $D - P_2$ by past returns, we consider a multiple regression of $\frac{D-P_2}{\sqrt{Var[D-P_2]}}$ on $\frac{P_1-P_0}{\sqrt{Var[P_1-P_0]}}$ and $\frac{P_2-P_1}{\sqrt{Var[P_2-P_1]}}$. The coefficients are denoted by α_1 and α_2 respectively. Figures 7-9 show that the introduction of technical trader reduces the magnitudes of α_1 and α_2. It is interesting to note that, as shown in section 4.3, because technical trader provider extra liquidity to the market, the liquidity premium demanded by the liquidity provider is partially reduced, P_2 moves back closer to the fair value, and the predictability of $D - P_2$ by $P_1 - P_0$ is thus reduced. The existence of technical trader reduces price predictability and enhances market efficiency.

[*Insert Figure 7-9 Here*]

It is noteworthy that when the liquidity providers are risk neutral, the successive price changes are uncorrelated in our setting because under the competition and risk neutral assumptions, the price equals the present value of future dividends conditional on liquidity providers' information set and the expected excess return based on their information set is zero. As a result, accumulated excess return follows a martingale process and the autocorrelation is zero.

[9] Overreaction and correction is another potential driver for short-term return reversal, Subrahmanyam (2005) studies the joint effect of inventory risk and beliefs reversion.

4.3 Trading Patterns of Technical Trader and Return Predictability of the Trade

As mentioned before, to better understand the return reversal we need to study the trading patterns of technical trader and informed trader. In this section we focus on the technical trader's strategy $Z = h_1\omega_1$ and Proposition 2 summarize the properties of technical trading.

Proposition 2 *Uninformed technical trader employ contrarian strategy and the trade has price impact and can positively forecast future return.*

$$Corr\left[Z, P_1 - P_0\right] = -1, \tag{4.6}$$

$$Corr\left[Z, P_2 - P_1\right] = \frac{h_1\left(3\tau_{11} + \lambda_{21} - 4\lambda_{11}\right)c_0}{4\sqrt{Var\left[Z\right]Var\left[P_2 - P_1\right]}}, \tag{4.7}$$

$$Corr\left[Z, D - P_2\right] = \frac{h_1^2\lambda_{22}c_0}{2\sqrt{Var\left[Z\right]Var\left[D - P_2\right]}}. \tag{4.8}$$

We start by point out that the information set that the uninformed investors have in the second period is equivalent to what the liquidity providers have in the first period. Since the liquidity provider is risk averse, she requires liquidity premium as compensation for holding undesired positions. As a result the price P_1 deviates from the fair value by an amount proportional to ω_1. For instance a positive ω_1 requires that the price to be higher than the fair value and the extent reflects the liquidity premium. The technical trader knows that by providing extra liquidity to the market, he can earn part of the liquidity premium. He thus chooses to trade in the opposite direction of ω_1 in period 2, as shown in (4.6). Technical trader's order incurs price impact, therefore Z is positively correlated with $P_2 - P_1$. The positive sign of (4.7) is confirmed in the numerical analysis. Section 4.4 shows that the informed trader's order in period 2 is also negatively related to ω_1 since he knows that the price is set above the fair value. As a result, the liquidity provider unloads part of her position in period 1. The price P_2 moves in the opposite direction of P_1. Hence, $P_1 - P_0$ and $P_2 - P_1$ are negatively correlated. Since the liquidity provider is risk averse, her expected profit must be proportional to her position in period 2, $D - P_2$, $P_1 - P_0$, and $P_2 - P_1$ are thus negatively correlated. Because both the technical trader and the informed trader take into account the impact of their trades on the price, the liquidity provider only unloads part of her initial position, $P_2 - E[D|\mathcal{F}_1]$ is still negatively related to ω_1. Hence, we have $Corr\left[Z, D - P_2\right] > 0$. This intuition shows that the technical trader trades as a contrarian, and his trade positively forecasts future short-horizon returns, which in turn implies that technical trader makes positive expected profit on positions taken in period 2. Panels A, B, and C of Figures 10-12 plot $Corr\,[Z, P_1 - P_0]$, $Corr\,[P_2 - P_1, Z]$, and $Corr\,[Z, D - P_2]$ against exogenous parameters σ_D, σ_U, and γ. Consistent with our explanation, the numerical analysis shows that $Corr\,[P_2 - P_1, Z] > 0$, and $Corr\,[Z, D - P_2] > 0$ hold for a wide range of parameters.

[*Insert Figures 10-12 Here*]

Empirical studies have shown that individuals tend to be negative feedback traders or contrarians, namely, they buy after prices go down and sell after prices go up, at least in the short term. Choe, Kho, and Stulz (1999) document short-horizon contrarian trading tendencies of Korean individual investors, Grinblatt and Keloharju (2000, 2001) report contrarian patterns (both long- and short-term) using Finnish data, Jackson (2003) show strong evidence of short-horizon patterns using Australian data, and Richards (2005) reports similar findings in six Asian emerging markets. In the U.S., Goetzmann and Massa (2002) investigate individuals who invest in an index fund and find that contrarians outnumber momentum traders two to one, and Griffin, Harris, and Topaloglu (2003) document a short-horizon contrarian tendency of traders who submit orders in NASDAQ stocks through a set of retail brokers. Because individual investors on average are less informative compared to institutional investors, they are more likely to adopt technical analysis in trading. If we interpret the technical trader in our model as individual investors, these empirical findings are consistent with our model. As emphasized in the beginning, the structure of our model can be used to address the short-horizon trading and asset pricing patterns.

In addition, our model can explain the empirical findings of the short-term predictability of individual investors. Jackson (2003) reports that the net flows of small investors positively predict future short-horizon returns in Australia. Kaniel, Saar, and Titman (2008) find that in the U.S. stocks experience statistically significant positive and negative excess return in one month after intense individual buying and selling respectively. Using signed small-trade volume as a proxy for individual trading in Taiwan, Barber, Odean, and Zhu (2009) show similar results on the return patterns in the several weeks after heavy buying and selling by individuals. Since the individual investors lack information advantage, it is not easy to understand why the trade of individual investors has short term forecasting ability. Our model shows that the forecasting power comes from implicit liquidity provision instead of information advantage. As explained in the section 2, our explanation enriches the models by Grossman and Miller (1998), Campbell, Grossman, and Wang (1994).

We consider two measures of expected profit to shed light on the predictability power of technical trading from a different perspective. The first measure

$$E[sign(Z)(D-P_2)]$$

is the expected profit of investing one share of stocks following the technical trader's trading, which represents a "directional bet" without taking into account the trade size of technical trader. The second measure is the expected profit of technical trader's total order size, defined as

$$E[Z(D-P_2)].$$

We explore the effects of fundamental risk σ_D, noise trader risk σ_U, and liquidity provider's risk aversion γ on the technical trader's predictability powers. Panels D and E in Figures 10-12 plot two profit measures against σ_D, σ_U, and γ, respectively. We find that an increase in σ_D, σ_U, or γ will lead to a larger $E[sign(Z)(D-P_2)]$ and a larger $E[Z(D-P_2)]$. Hence, technical trading is more profitable when the fundamental risk is larger, when the noise trader risk is larger, or when the liquidity providers become more risk averse. Intuitively, an increase in σ_D, σ_U, or γ will cause the price to be more predictable (measured by return autocorrelation). Kaniel, Saar, and Titman (2008) find that individual investors' order has more forecasting power for small stock. Small stocks usually involve larger fundamental risk and less liquidity. Hence, our theoretical predictions are consistent with their empirical evidence.

4.4 Trading Patterns of Informed Trader and Return Predictability of the Trades

In this section, we explore the trading patterns of the informed trader. Recall his trade in period 1 $X_1 = \beta_{11}D$ is similar to the period 1 trade in Kyle (1985). His trade in period 2 is more interesting: $X_2 = \beta_{21}D + \beta_{22}(X_1 + U_1) + \beta_{23}Z$. Simply calculation shows that $X_2 = \beta_{21}(D - E_1[D]) + \frac{1}{2}h_1(X_1 + U_1)$. Hence, his trade consists of two components. The first part $\beta_{21}(D - E_1[D])$ represents the "informational part", which is proportional to the residual information. This component is very typical in the literature. The second part $\frac{1}{2}h_1(X_1 + U_1)$ is the "non-informational part", which is related to the price impact due to inventory effect. Intuitively, the informed trader trades less aggressively to take advantage of his private information due to larger price impact. Proposition 3 characterizes the trading strategies and return predictability of the informed trades.

Proposition 3 *In the benchmark economy, the informed trader employs contrarian strategy in* *period 2; the trades have price impact and can positively forecast future return:*

$$Corr\left[X_1, P_2 - P_1\right] = \frac{\left[2 + \left(\tau_{11} + \lambda_{21} - 4\lambda_{11}\right)\beta_{11}\right]\beta_{11}\sigma_D^2}{4\sqrt{Var\left[X_1\right]Var\left[P_2 - P_1\right]}} \tag{4.9}$$

$$Corr\left[X_1, D - P_2\right] = \frac{\left[2 - \left(\tau_{11} + \lambda_{21}\right)\beta_{11}\right]\sigma_D,}{4\sqrt{Var\left[X_1\right]Var\left[D - P_2\right]}} \tag{4.10}$$

$$Corr\left[X_2, P_1 - P_0\right] = \frac{h_1\lambda_{11}c_0}{2\sqrt{Var\left[X_2\right]Var\left[P_1 - P_0\right]}}, \tag{4.11}$$

$$Corr\left[X_2, D - P_2\right] = \frac{\left[4\sigma_D^2 - \left(\tau_{11} + \lambda_{21}\right)\left(3\tau_{11} - \lambda_{21}\right)c_0\right]\beta_{21}}{8\sqrt{Var\left[X_2\right]Var\left[D - P_2\right]}}, \tag{4.12}$$

The correlations between the informed trades and the technical trade satisfy

$$Corr\left[X_1, Z\right] = \frac{h_1\beta_{11}^2\sigma_D^2}{\sqrt{Var\left[X_1\right]Var\left[Z\right]}},$$

$$Corr\left[X_2, Z\right] = \frac{h_1^2c_0}{2\sqrt{Var\left[X_2\right]Var\left[Z\right]}}.$$

It is worth noting that the informed trader also employs contrarian strategy in period 2, as $Cov\left[X_2, P_1 - P_0\right] < 0$. It is easy to show that when the liquidity provider is risk neutral as in Kyle model, past price is useless. The trade of the informed trader is independent of $P_1 - P_0$ and he trades only on the residual information $D - P_1 = D - E_1[D]$. In a two-period competitive trading model with risk neutral market maker, Hirshleifer, Subrahmanyam, and Titman (1994) show that only early informed traders employ contrarian strategy but it is unprofitable on average, and the late-informed traders are neither trend chaser nor contrarian. It is worth noting that the trading patterns of the technical trader and the informed trader in our model share the same properties: they adopt contrarian trading strategy in the second period, their trades have positive price impact on the price, and their trades positively forecasts futures returns. However, as discussed before, they trade for different purposes: the informed trader trades because of his informational advantage whereas the technical trader trades for the purpose of liquidity provision. The implications for empirical studies is that it is not enough to study only the lead-lag or contemporaneous relationships between trades and return to determine whether the trade is informed or uninformed.

5 Extensions

In the section we study several extensions of the benchmark model to see if the main results are robust or to what extent they will be modified. While remain the assumption on informed trader and liquidity provider intact, we first consider a variant in which the risk neutral technical trader is replaced with a risk averse one. We next consider the imperfect competition among multiple risk neutral technical traders.

5.1 A Risk Averse Technical Trader

Assume a risk averse technical trader has a CARA utility function of $-\exp\left(-\gamma_z W_3^Z\right)$, where γ_z denotes his risk aversion coefficient. We solve the technical trader's optimization problem following the procedure outlined in the section 2.2. Note that Lemma 2 still holds and the solutions to informed and liquidity trader's problems are unchanged, where the technical trader's optimal trade in period 2 is now given by

$$Z = h_1\omega_1 = \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22} + 2\gamma_z Var[D - P_2|\mathcal{F}_1]}\omega_1. \tag{5.1}$$

Compared with the solution in section 2.2, it is clear that the risk-averse technical trader trades less aggressively. Taking together the optimal trading of the informed trader and liquidity provider, and market clearing conditions in section 2, a linear equilibrium is characterized in the following.

Theorem 4 *With a risk averse technical trader in the benchmark economy, a linear equilibrium is the same as those given in Theorem 1 except that h_1 is the root to the equation*

$$h_1 = \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22} + 2\gamma_z Var[D - P_2|\mathcal{F}_1]},$$

where

$$Var[D - P_2|\mathcal{F}_1] = \sigma_D^2 + \begin{pmatrix} \lambda_{21} - 2\tau_{21} & \lambda_{22} - 2\tau_{22} \end{pmatrix} \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix} + \left(\frac{\tau_{11} - \lambda_{21}}{4}\right)^2 c_0.$$

5.2 Multiple Risk Neutral Technical Traders

We introduce n risk neutral technical traders in another extension. The trading game becomes more involved as a technical trader now has to consider the strategic responses of all other technical traders, in addition to those of informed and liquidity providers. We denote each technical trader's trade in period 2 by Z_i and conjecture that each adopts a linear trading strategy

$$Z_i = h_i \omega_1 \text{ for } i \in \{1, \cdots, n\}.$$

Note that in equilibrium, each technical trader chooses same trading intensity h_i and same trade. Let $h_1 = nh_i$ and

$$Z = \sum_{i=1}^{n} Z_i = nZ_i = h_1 \omega_1$$

to be the total order flow submitted by n technical traders. The linear equilibrium is characterized as follows.

Theorem 5 *When there are n risk neutral technical traders in the benchmark economy, a linear equilibrium is given by*

$$\begin{aligned}
& X_1 = \beta_{11} D, \qquad X_2 = \beta_{21} D + \beta_{22} (X_1 + U_1) + \beta_{23} Z, \\
& Z_i = h_i (X_1 + U_1), \\
& P_1 = \lambda_{11} (X_1 + U_1), \qquad P_2 = \lambda_{21} (X_1 + U_1) + \lambda_{22} (X_2 + Z + U_2)
\end{aligned}$$

where

$$\begin{aligned}
\beta_{11} &= \frac{2(n+1)^2 \lambda_{22} - (n+1)(\lambda_{21} + n\tau_{11})}{4(n+1)^2 \lambda_{11}\lambda_{21} - (\lambda_{21} + n\tau_{11})^2}, \\
h_i &= \frac{\tau_{11} - \lambda_{21}}{(n+1)\lambda_{22}}, \\
\lambda_{11} &= \frac{\lambda_{21} + (2n+1)\tau_{11}}{2(n+1)} + \gamma Var[P_2|\mathcal{F}_1].
\end{aligned}$$

and β_{21}, β_{22}, β_{23}, λ_{21}, λ_{22}, τ_{11}, τ_{21}, τ_{22}, c_0, c_1, c_2 have the same expressions as those given in Theorem 1.

We focus on the issues of predictability and profitability. In our view, academic work actively searches for return predictability of technical analysis but few attentions are devoted to its profitability. But conventional industrial view is if more people trading on a strategy, the profitability and predictability both decline to zero by competition. Proposition 4 confirms the view.

Proposition 4 *When there are n risk neutral technical traders in the benchmark economy, we* *obtain*

$$Cov\left[Z_i, D - P_2\right] = \frac{1}{2}\lambda_{22}h_i^2 c_0,$$
$$Cov\left[Z, D - P_2\right] = \frac{1}{2n}\lambda_{22}h_1^2 c_0.$$

When n goes to infinity,

$$\beta_{11} \to \frac{2\lambda_{22} - \tau_{11}}{4\lambda_{11}\lambda_{21} - \tau_{11}^2}$$
$$c_0 \to \beta_{11}^2\sigma_D^2 + \sigma_U^2$$
$$h_i \to 0$$
$$h_1 \to \frac{\tau_{11} - \lambda_{21}}{\lambda_{22}}$$

Both $Cov\left[Z_i, D - P_2\right]$ and $Cov\left[Z, D - P_2\right]$ go to zero.

6 Conclusion

In this paper we build a strategic trading model to capture short-horizon trading of informed and uninformed traders when the liquidity providers are risk averse. In our setup both the informed and uninformed traders employ technical analysis. We show that when traders exploit information contained in historical prices or volumes, they employ contrarian strategy and provide extra implicit liquidity in addition to the explicit liquidity provided by the liquidity providers. Second, we find that their trades can be used to predict the stock returns in near future. Consistent with some recent empirical studies on individual investor trading, our analysis extends the existing theoretical understanding of short-term trading and asset pricing patterns. We also show technical trading reduces price predictability and enhances market efficiency by improving the price discovery process, increasing price informativeness, and reducing price impact and price volatility.

Our model has broader implications on some hotly debated issues. For instance, the rise of high-frequency trading (HFT) in the past several years, which usually involve buying and selling stock by a computer algorithm based on past prices and volumes and holding stocks for only a short period time, has increased the attention of investors as well as regulators. HFT accounts for a large proportion of US equity trading volume and earns a annual trading profit about \$2.8 billion in the U.S. Our model demonstrates technical trading in the short run mainly provide implicit liquidity to the markets and make market more efficient. Hence, the critiques regarding HFT, including that HFTs systematically anticipate and trade in front of non-HFTs, flee in volatile times, and are detrimental to the markets, may not be valid. In fact, consistent with our theoretical predictions, Brogaard (2010) reports that HFT tend to follow a price reversal strategy driven by order imbalances, HFTs do not seem to systematically engage in a non-HFTs anticipatory trading strategy, HFTs add substantially to the price discovery process HFTs may dampen intraday volatility so HFT tends to improve market quality.

A Appendix

Preliminaries. First, we compare the derived trading strategies with the conjectured strategies.

For the informed trader's maximization problem, comparing (3.12) with (3.6), we obtain

$$\beta_{20} = -\frac{\lambda_{20}}{2\lambda_{22}}, \quad \beta_{21} = \frac{1}{2\lambda_{22}}, \quad \beta_{22} = -\frac{\lambda_{21}}{2\lambda_{22}}, \quad \beta_{23} = -\frac{1}{2}. \tag{A.1}$$

Equating the coefficients of (3.13) and (3.5) delivers

$$\beta_{10} = \frac{[2\lambda_{22}(\beta_{20} + \beta_{23}h_0)(\beta_{22} + \beta_{23}h_1) - \lambda_{10}]}{2\lambda_{11} - 2\lambda_{22}(\beta_{22} + \beta_{23}h_1)^2}, \tag{A.2}$$

$$\beta_{11} = \frac{1 + 2\lambda_{22}\beta_{21}(\beta_{22} + \beta_{23}h_1)}{2\lambda_{11} - 2\lambda_{22}(\beta_{22} + \beta_{23}h_1)^2}. \tag{A.3}$$

For the technical trader's maximization problem, plugging (A.1) into (3.15) yields

$$Z = \frac{(\tau_{10} - \lambda_{20}) + (\tau_{11} - \lambda_{21})\,\omega_1}{2\lambda_{22}}. \tag{A.4}$$

Equating the coefficients of (A.4) and (3.7) delivers

$$h_0 = \frac{\tau_{10} - \lambda_{20}}{2\lambda_{22}}, \tag{A.5}$$

$$h_1 = \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22}}. \tag{A.6}$$

For the liquidity provider's maximization problem, using the market clearing conditions (3.3) and (3.4), we obtain from (3.17)

$$\lambda_{20} = \tau_{20}, \tag{A.7}$$

$$\lambda_{21} = \tau_{21} + \gamma Var\left[D|\mathcal{F}_2\right], \tag{A.8}$$

$$\lambda_{22} = \tau_{22} + \gamma Var\left[D|\mathcal{F}_2\right]. \tag{A.9}$$

We also obtain from (3.19)

$$\lambda_{10} = \lambda_{20} + \lambda_{22}\left[\beta_{20} + \beta_{21}\tau_{10} + (1+\beta_{23})\,h_0\right] \tag{A.10}$$

$$\lambda_{11} = \lambda_{21} + \lambda_{22}\left[\beta_{21}\tau_{11} + \beta_{22} + (1+\beta_{23})\,h_1\right] + \gamma Var\left[P_2|\mathcal{F}_1\right] \tag{A.11}$$

Plugging (A.5) and (A.6) into (A.2) and (A.3) yields

$$\beta_{10} = \frac{(\tau_{10}+\lambda_{20})(\tau_{11}+\lambda_{21}) - 8\lambda_{10}\lambda_{22}}{16\lambda_{11}\lambda_{22} - (\tau_{11}+\lambda_{21})^2}, \tag{A.12}$$

$$\beta_{11} = \frac{8\lambda_{22} - 2\,(\tau_{1,1}+\lambda_{21})}{16\lambda_{11}\lambda_{22} - (\tau_{11}+\lambda_{21})^2}. \tag{A.13}$$

Note that, plugging (A.1), (A.5) and (A.6) into (A.10) and (A.11), we obtain

$$\lambda_{10} = \frac{3\tau_{10}+\lambda_{20}}{4} \tag{A.14}$$

$$\lambda_{11} = \frac{3\tau_{11}+\lambda_{21}}{4} + \gamma Var\left[P_2|\mathcal{F}_1\right] \tag{A.15}$$

These results are useful in the following proofs. ■

Proof of Lemma 1. We have

$$\begin{aligned}
a &= Cov\left[P_2 - P_1, \left.\frac{(E[D|\mathcal{F}_2]-P_2)(D-P_2)}{\gamma Var\left[D|\mathcal{F}_2\right]}\right|\mathcal{F}_1\right] \\
&= Cov\left[P_2 - P_1, \left.\frac{(E[D|\mathcal{F}_2]-P_2)^2}{\gamma Var\left[D|\mathcal{F}_2\right]}\right|\mathcal{F}_1\right] \\
&= \gamma Var\left[D|\mathcal{F}_2\right] Cov\left[(\lambda_{21}-\lambda_{11})\omega_1 + \lambda_{22}\omega_2, (\omega_1+\omega_2)^2\right],
\end{aligned} \tag{A.16}$$

where the second equality follows from the law of iterated expectation, and the third equality comes from plugging in (3.1), (3.2), (3.9) and using the results (A.7)-(A.9). Because $E[D] = E[U_1] = E[U_2] = 0$, and these normally distributed random variables are independent from each other, we obtain

$$Cov\left[\omega_1, \omega_1^2\right] = E\left[\omega_1^3\right] = E\left[(\beta_{11}D + U_1)^3\right] = 0,$$

because the third moment or the skewness of normally distributed random variable with zero mean is zero. Similarly, we have $Cov\left[\omega_2, \omega_2^2\right] = 0$. In addition,

$$\begin{aligned}
Cov\left[\omega_2, \omega_1\omega_2\right] &= Cov\left[\omega_1, \omega_2^2\right] \\
&= Cov\left[\beta_{11}D + U_1, \left[\beta_{21}D + (\beta_{22} + (1+\beta_{23})\,h_1)\,(\beta_{11}D + U_1) + U_2\right]^2\right] \\
&= 0
\end{aligned}$$

Similarly, $Cov\left[\omega_1, \omega_1\omega_2\right] = Cov\left[\omega_2, \omega_1^2\right] = 0$. Plugging these results in (A.16) yields $a = 0$. ■

Proof of Lemma 2. Because $D_0 = 0$, we conjecture that $\tau_{10} = \tau_{20} = 0$, then we obtain $\lambda_{20} = 0$ from (A.7), $\beta_{20} = 0$ from (A.1), $h_0 = 0$ from (A.5), $\lambda_{10} = 0$ from (A.14), and $\beta_{10} = 0$ from (A.12) sequentially. Therefore we get $\tau_{10} = 0$ from (3.21) and $\tau_{20} = 0$ from (3.26) which verify that our initial conjecture on τ_{10} and τ_{20} are correct. ■

Proof of Theorem 1. Given Lemma 2 and the results in the Preliminary, we only need to apply the projection theorem to obtain the expressions for $Var\left[P_2|\mathcal{F}_1\right]$ and $Var\left[D|\mathcal{F}_2\right]$, and note that the SOC in (3.14) can be rewritten as $16\lambda_{11}\lambda_{22} < \left(\tau_{11} + \lambda_{21}\right)^2$. ■

Proof of Theorem 2. First, we must have $\beta_{23} = 0$ and $h_1 = 0$. Solutions to the informed trader's maximization problem, given in (3.12) and (3.13) reduce to

$$X_2 = \frac{D - \lambda_{21}\omega_1}{2\lambda_{22}},$$
$$X_1 = \frac{\left(1 + 2\lambda_{22}\beta_{21}\beta_{22}\right) D}{2\lambda_{11} - 2\lambda_{22}\beta_{22}^2}.$$

and SOCs are

$$\lambda_{11} < \lambda_{22}\beta_{22}^2 \quad \text{and} \quad \lambda_{22} > 0.$$

Equating the coefficient with the conjectured trading strategies yields

$$\beta_{21} = \frac{1}{2\lambda_{22}}, \quad \beta_{22} = -\frac{\lambda_{21}}{2\lambda_{22}},$$
$$\beta_{11} = \frac{1 + 2\lambda_{22}\beta_{21}\beta_{22}}{2\lambda_{11} - 2\lambda_{22}\beta_{22}^2} = \frac{2\lambda_{22} - \lambda_{21}}{2\lambda_{11}\lambda_{22} - \lambda_{21}^2}.$$

Liquidity provider's maximization problem is similar to (3.18) and (3.19), the market clearing conditions and equilibrium pricing functions yield (A.8), (A.9) and

$$\lambda_{11} = \lambda_{21} + \lambda_{22}\left(\beta_{21}\tau_{11} + \beta_{22}\right) + \gamma Var\left[P_2|\mathcal{F}_1\right].$$

which can be re-written as

$$\lambda_{11} = \frac{\tau_{11} + \lambda_{21}}{2} + \gamma Var\left[P_2|\mathcal{F}_1\right]$$

The liquidity provider's beliefs updating problem is unchanged except that $h_1 = 0$. ■

Proof of Theorem 3. When liquidity provider is risk neutral, her role is exactly the same as the role of the market maker in Kyle (1985). The equilibrium can be derived directly from the Theorem 2 of Kyle (1985). In fact, Huddart, Hughes, and Levine (2001) provide the solution. ■

Proof of Proposition 1. Let $E_t\left[\cdot\right] = E\left[\cdot|\mathcal{F}_t\right]$. Using the equilibrium conditions given in Theorem

1, we obtain

$$
\begin{aligned}
E_1[\omega_2] &= E_1\left[\beta_{21} D + \left(\beta_{22} + (1+\beta_{23})\, h_1\right) \omega_1 + U_2\right] \\
&= \left(\beta_{21}\tau_{11} + \beta_{22} + \frac{1}{2} h_1\right) \omega_1 \\
&= \frac{3\left(\tau_{11} - \lambda_{21}\right)}{4\lambda_{22}} \omega_1
\end{aligned}
\tag{A.17}
$$

and applying the law of iterated expectation, we derive

$$
\begin{aligned}
Cov\left[P_1 - P_0, P_2 - P_1\right] &= E\left[\lambda_{11}\omega_1\left(\left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22} E_1[\omega_2]\right)\right] \\
&= \lambda_{11}\left[\lambda_{21} - \lambda_{11} + \frac{3\left(\tau_{11} - \lambda_{21}\right)}{4}\right] \sigma^2_{\omega_1} \\
&= \frac{1}{4}\lambda_{11}\left(3\tau_{11} + \lambda_{21} - 4\lambda_{11}\right) c_0, \\
Cov\left[P_2 - P_1, D - P_2\right] &= E\left[\left(P_2 - P_1\right) E_2\left[D - P_2\right]\right] \\
&= -\gamma Var_2[D]\, E\left[\left(\omega_1 + \omega_2\right)\left(P_2 - P_1\right)\right] \\
&= \left(\tau_{21} - \lambda_{21}\right) E\left[\left(\omega_1 + \omega_2\right)\left[\left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22}\omega_2\right]\right] \\
&= \left(\tau_{21} - \lambda_{21}\right) \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix} \begin{pmatrix} \lambda_{21} - \lambda_{11} \\ \lambda_{22} \end{pmatrix}, \\
Cov\left[P_1 - P_0, D - P_2\right] &= E\left[\lambda_{11}\omega_1\left(\left(\tau_{11} - \lambda_{21}\right)\omega_1 - \lambda_{22} E_1[\omega_2]\right)\right] \\
&= \lambda_{11}\left(\tau_{11} - \lambda_{21} - \frac{3\left(\tau_{11} - \lambda_{21}\right)}{4}\right) \sigma^2_{\omega_1} \\
&= \frac{1}{2}\lambda_{11}\lambda_{22} h_1 c_0.
\end{aligned}
$$

Note that these covariances can be derived directly using the equilibrium conditions, but the application of the law of iterated expectation often yield the results more easily.

Using the equilibrium conditions given in Theorem 2, we obtain

$$
\begin{aligned}
E_1[\omega_2] &= E_1\left[\beta_{21} D + \beta_{22}\omega_1 + U_2\right] \\
&= \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22}} \omega_1,
\end{aligned}
$$

and applying the law of iterated expectation, we derive

$$
\begin{aligned}
Cov\left[P_1 - P_0, P_2 - P_1\right] &= Cov\left[\lambda_{11}\omega_1, \left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22}\left(\beta_{21}D + \beta_{22}\omega_1\right)\right] \\
&= \lambda_{11}\left[\lambda_{21} - \lambda_{11} + \lambda_{22}E_1\left[\omega_2\right]\right]\sigma^2_{\omega_1} \\
&= \frac{1}{2}\lambda_{11}\left(\lambda_{21} - 2\lambda_{11} + \tau_{11}\right)d_0, \\
Cov\left[P_2 - P_1, D - P_2\right] &= \left(\tau_{21} - \lambda_{21}\right)\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} d_0 & d_1 \\ d_1 & d_2 \end{pmatrix}\begin{pmatrix} \lambda_{21} - \lambda_{11} \\ \lambda_{22} \end{pmatrix}, \\
Cov\left[P_1 - P_0, D - P_2\right] &= Cov\left[\lambda_{11}\omega_1, D - \lambda_{21}\omega_1 - \lambda_{22}\omega_2\right] \\
&= \lambda_{11}\left(\tau_{11} - \lambda_{21} - \frac{\tau_{11} - \lambda_{21}}{2}\right)\sigma^2_{\omega_1} \\
&= \frac{1}{2}\lambda_{11}\left(\tau_{11} - \lambda_{21}\right)d_0.
\end{aligned}
$$

Using the equilibrium conditions given in Theorem 3, direct calculation yields

$$
\begin{aligned}
Cov\left[P_1 - P_0, P_2 - P_1\right] &= \frac{1 - \lambda_{11}\beta_{11}}{2}\lambda_{11}\beta_{11}\sigma^2_D - \frac{\lambda^2_{11}}{2}\sigma^2_U = 0, \\
Cov\left[P_2 - P_1, D - P_2\right] &= \left(\frac{1 - \lambda_{11}\beta_{11}}{2}\right)^2\sigma^2_D + \left(\frac{\lambda_{11}}{2}\right)^2\sigma^2_U - \lambda^2_{22}\sigma^2_U = 0, \\
Cov\left[P_1 - P_0, D - P_2\right] &= \frac{1 - \lambda_{11}\beta_{11}}{2}\lambda_{11}\beta_{11}\sigma^2_D - \frac{\lambda^2_{11}}{2}\sigma^2_U = 0.
\end{aligned}
$$

Finally, the correlations can be derived after direct calculation of $Var\left[P_1 - P_0\right]$, $Var\left[P_2 - P_1\right]$ and $Var\left[D - P_2\right]$. ■

Proof of Proposition 2. We directly obtain

$$
Cov\left[Z, P_1 - P_0\right] = Cov\left[h_1\omega_1, \lambda_{11}\omega_1\right] = h_1\lambda_{11}c_0.
$$

Applying the law of iterated expectation and using (A.17) yield

$$
\begin{aligned}
Cov\left[Z, P_2 - P_1\right] &= Cov\left[h_1\omega_1, \left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22}E_1\left[\omega_2\right]\right] \\
&= h_1\left[\lambda_{21} - \lambda_{11} + \lambda_{22}E_1\left[\omega_2\right]\right]\sigma^2_{\omega_1} \\
&= \frac{1}{4}h_1\left(3\tau_{11} + \lambda_{21} - 4\lambda_{11}\right)c_0, \\
Cov\left[Z, D - P_2\right] &= Cov\left[h_1\omega_1, \left(\tau_{11} - \lambda_{21}\right)\omega_1 - \lambda_{22}E_1\left[\omega_2\right]\right] \\
&= \frac{1}{4}h_1\left(\tau_{11} - \lambda_{21}\right)\sigma^2_{\omega_1} \\
&= \frac{1}{2}h^2_1\lambda_{22}c_0.
\end{aligned}
$$

where, as before, $E_1\left[\omega_2\right] = E\left[\omega_2 | \mathcal{F}_1\right]$. The correlations can be calculated accordingly. ■

Proof of Proposition 3. Using the equilibrium conditions given in Theorem 1, we obtain

$$\begin{aligned}
Cov\left[X_1, P_2 - P_1\right] &= Cov\left[\beta_{11} D, \left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22}\left[\beta_{21} D + \left[\beta_{22} + \left(1+\beta_{23}\right) h_1\right]\omega_1 + U_2\right]\right] \\
&= \left[\left(\lambda_{21} - \lambda_{11}\right)\beta_{11} + \lambda_{22}\left[\beta_{21} + \left[\beta_{22} + \left(1+\beta_{23}\right) h_1\right]\beta_{11}\right]\right]\beta_{11}\sigma_D^2 \\
&= \left(\frac{1}{2} + \frac{\tau_{11} + \lambda_{21} - 4\lambda_{11}}{4}\beta_{11}\right)\beta_{11}\sigma_D^2, \\
Cov\left[X_1, D - P_2\right] &= \beta_{11}\sigma_D^2 - Cov\left[\beta_{11} D, \lambda_{21}\omega_1 + \lambda_{22}\left[\beta_{21} D + \left(\beta_{22} + \left(1+\beta_{23}\right) h_1\right)\omega_1 + U_2\right]\right] \\
&= \beta_{11}\sigma_D^2 - \lambda_{22}\beta_{11}\beta_{21}\sigma_D^2 - \beta_{11}^2\left(\lambda_{21} + \lambda_{22}\left(\beta_{22} + \left(1+\beta_{23}\right) h_1\right)\right)\sigma_D^2 \\
&= \left(\frac{1}{2} - \frac{\tau_{11} + \lambda_{21}}{4}\beta_{11}\right)\beta_{11}\sigma_D^2,
\end{aligned}$$

and

$$\begin{aligned}
Cov\left[X_2, P_1 - P_0\right] &= Cov\left[\beta_{21} D + \left(\beta_{22} + \beta_{23} h_1\right)\omega_1, \lambda_{11}\omega_1\right] \\
&= \lambda_{11}\left[\beta_{21}\tau_{11} + \beta_{22} + \beta_{23} h_1\right]\sigma_{\omega_1}^2 \\
&= \frac{1}{2}\lambda_{11} h_1 c_0, \\
Cov\left[X_2, D - P_2\right] &= Cov\left[\beta_{21} D + \left(\beta_{22} + \beta_{23} h_1\right)\omega_1, D - P_2\right] \\
&= \beta_{21} Cov\left[D, D - P_2\right] + \left(\beta_{22} + \beta_{23} h_1\right) E\left[E_1\left[D - P_2\right]\omega_1^2\right] \\
&= \frac{1}{2\lambda_{22}}\left[\frac{1}{2} - \frac{\tau_{11} + \lambda_{21}}{4}\beta_{11}\right]\sigma_D^2 - \frac{\tau_{11} + \lambda_{21}}{4\lambda_{22}}\frac{\tau_{11} - \lambda_{21}}{4}\sigma_{\omega_1}^2 \\
&= \frac{\beta_{21}}{2}\sigma_D^2 - \frac{\beta_{21}\left(\tau_{11} + \lambda_{21}\right)\left(3\tau_{11} - \lambda_{21}\right)}{8} c_0.
\end{aligned}$$

The calculation of $Cov\left[X_1, Z\right]$ and $Cov\left[X_2, Z\right]$ is straightforward. ∎

Proof of Theorem 4. Following the procedure given in section 2, solutions to the informed trader and the liquidity provider are unchanged. The risk averse technical trader maximizes his expected profits by choosing

$$Z = \frac{\tau_{11} - \lambda_{21}}{2\lambda_{22} + 2\gamma_z Var[D - P_2|\mathcal{F}_1]}\omega_1.$$

A direct application of the project theorem to $Var[D - P_2|\mathcal{F}_1]$ yields the its expression in the main text. ∎

Proof of Theorem 5. The derivation closely follows the procedure outlined in section 2 and preliminary. For informed trader, the maximization problem deliver (A.1), (A.3). For each technical trader i, his maximization problem in period 2 is given by

$$\max_{Z_i} E[W_3^{Z_i}|\mathcal{F}_1] = Z_i\left[\tau_{11}\omega_1 - \left(\lambda_{21}\omega_1 + \lambda_{22} E[X_2|\mathcal{F}_1] + \lambda_{22} Z_i + \lambda_{22}\left(n-1\right) Z_j\right)\right].$$

The FOC with respect to Z_i yields

$$\tau_{11}\omega_1 - (\lambda_{21}\omega_1 + \lambda_{22}E[X_2|\mathcal{F}_1] + \lambda_{22}Z_i + \lambda_{22}(n-1)Z_j) - \lambda_{22}(1+\beta_{23})Z_i = 0.$$

In equilibrium we have $Z_i = Z_j$ and

$$Z_i = \frac{[\tau_{11} - \lambda_{21} - \lambda_{22}(\beta_{21}\tau_{11} + \beta_{22})]\omega_1}{(n+1)\lambda_{22}(1+\beta_{23})}.$$

Plugging (A.1) into the above equation yields

$$Z_i = \frac{(\tau_{11} - \lambda_{21})\omega_1}{(n+1)\lambda_{22}}.$$

We thus obtain

$$h_i = \frac{\tau_{11} - \lambda_{21}}{(n+1)\lambda_{22}}. \tag{A.18}$$

Plugging $h_1 = nh_i$ into (A.3) yields

$$\beta_{11} = \frac{2(n+1)^2\lambda_{22} - (n+1)(\lambda_{21} + n\tau_{11})}{4(n+1)^2\lambda_{11}\lambda_{21} - (\lambda_{21} + n\tau_{11})^2}.$$

No change is made to the liquidity provider's maximization and beliefs updating problems except that $h_1 = nh_i$, thus (A.11) can be rewritten as

$$\lambda_{11} = \frac{\lambda_{21} + (2n+1)\tau_{11}}{2(n+1)} + \gamma Var[P_2|\mathcal{F}_1]$$

by substituting in (A.1) and (A.18). ■

Proof of Proposition 4. Using the equilibrium conditions in Theorem 5, we obtain

$$\begin{aligned}
E_1[\omega_2] &= E_1[\beta_{21}D + \beta_{22}\omega_1 + (1+\beta_{23})Z + U_2] \\
&= [\beta_{21}\tau_{11} + \beta_{22} + (1+\beta_{23})h_1]\omega_1 \\
&= \frac{(2n+1)(\tau_{11} - \lambda_{21})}{2(n+1)\lambda_{22}}\omega_1,
\end{aligned}$$

and

$$\begin{aligned}
Cov[Z_i, D - P_2] &= Cov[h_i\omega_1, E_1[D - P_2]] \\
&= Cov[h_i\omega_1, (\tau_{11} - \lambda_{21})\omega_1 - \lambda_{22}E_1[\omega_2]] \\
&= h_i\frac{(\tau_{11} - \lambda_{21})}{2(n+1)}\sigma^2_{\omega_1} \\
&= \frac{1}{2}\lambda_{22}h_i^2 c_0.
\end{aligned}$$

$Cov\,[Z, D - P_2]$ follows directly from $Z = nZ_i$. ■

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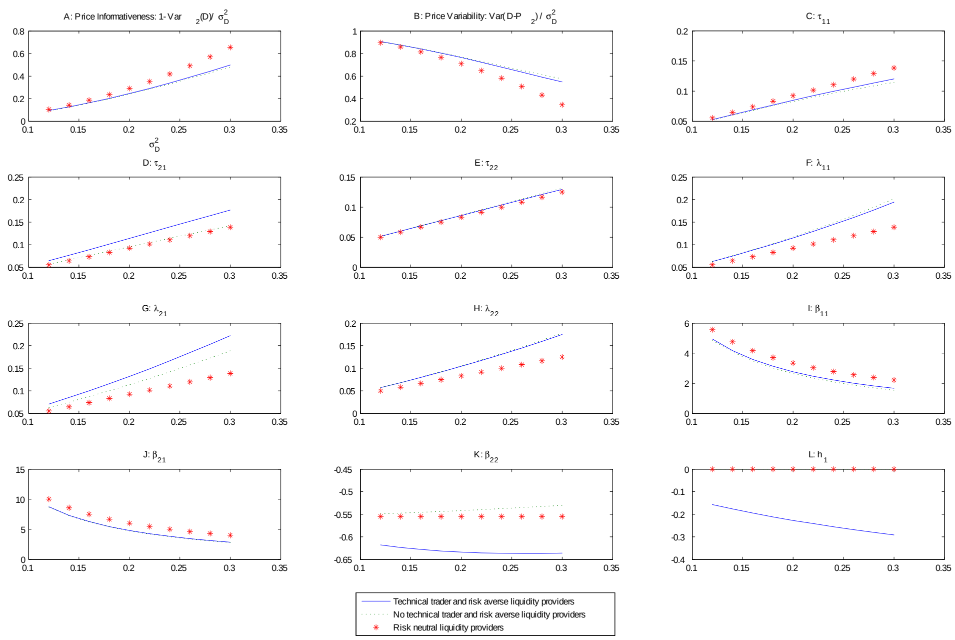

Figure 1: The effects of fundamental risk σ_D on the price informative, price variability, price impact, market liquidity and trading intensities ($\sigma_U = 1$, $\gamma = 1$).

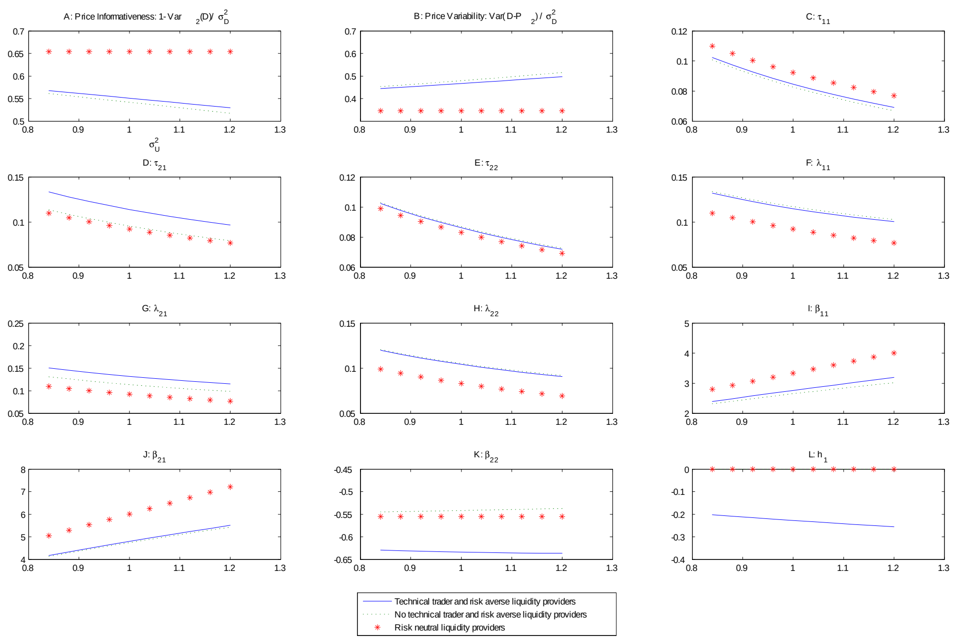

Figure 2: The effects of noise risk σ_U on the price informative, price variability, price impact, market liquidity and trading intensities ($\sigma_D = 0.2$, $\gamma = 1$).

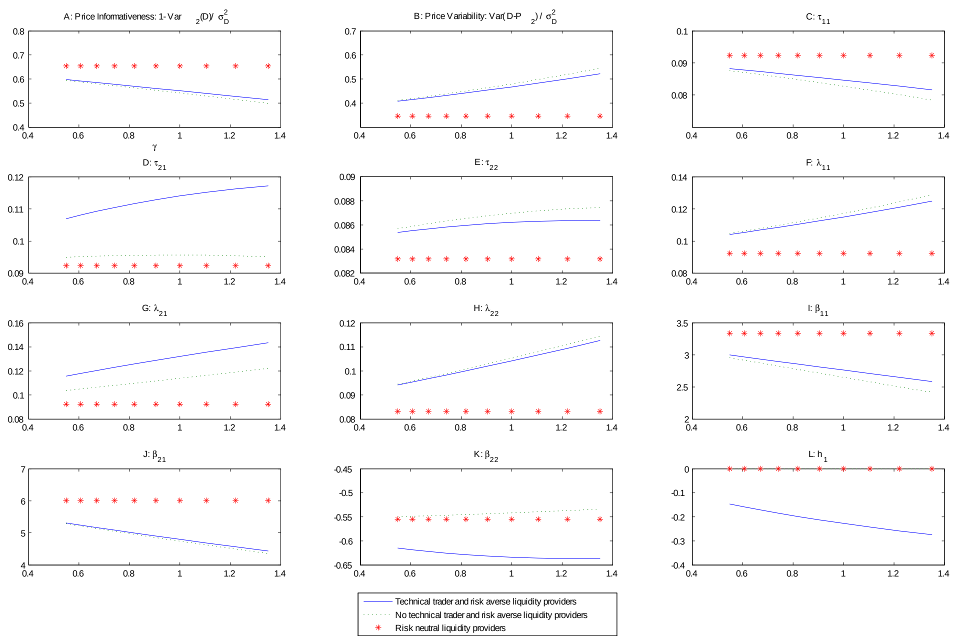

Figure 3: The effects of risk aversion γ on the price informative, price variability, price impact, market liquidity and trading intensities ($\sigma_D = 1, \sigma_U = 1$).

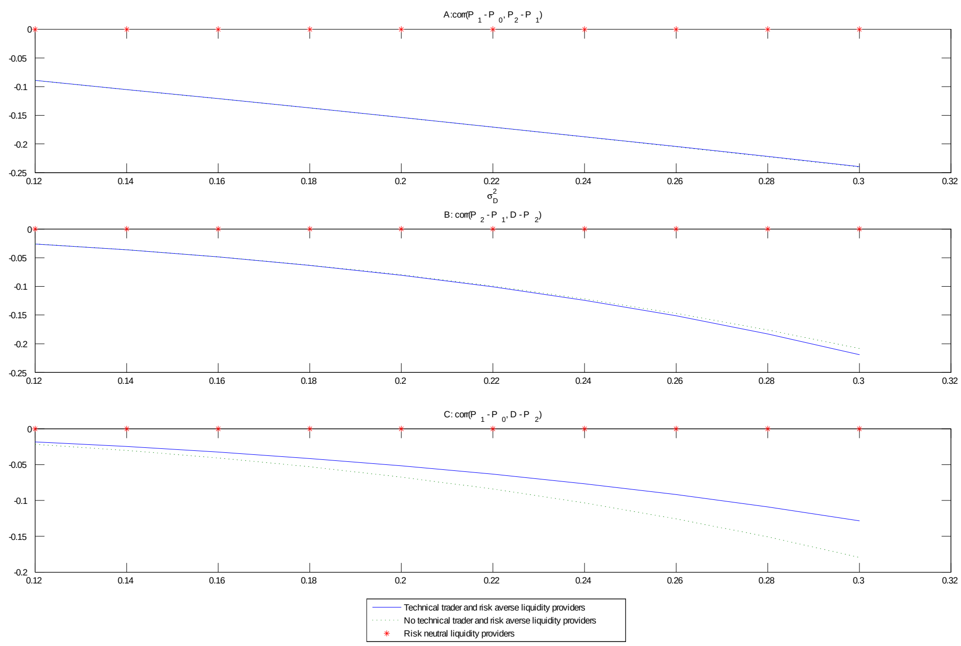

Figure 4: The effects of fundamental risk σ_D on return autocorrelation ($\sigma_U = 1$, $\gamma = 1$).

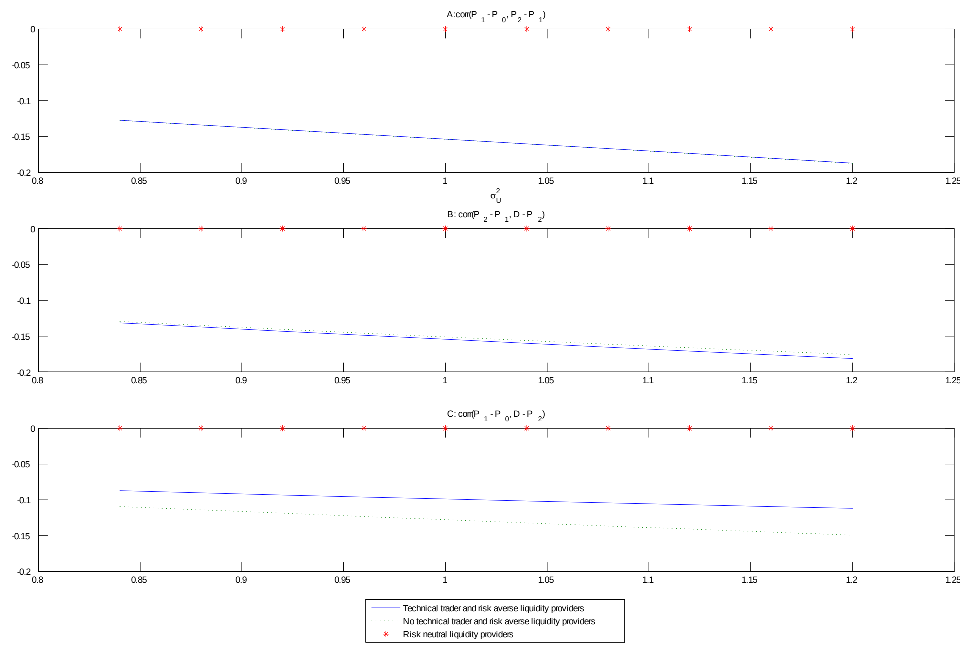

Figure 5: The effects of noise trader risk σ_U on return autocorrelation ($\sigma_D = 0.2$, $\gamma = 1$).

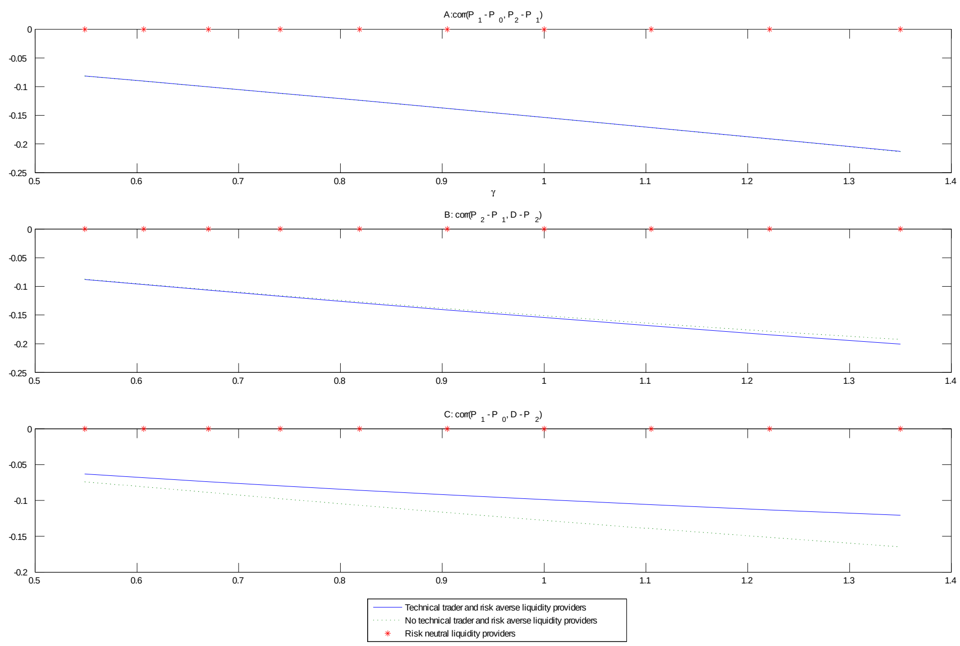

Figure 6: The effects of risk aversion γ on return autocorrelation ($\sigma_D = 0.2$, $\sigma_U = 1$).

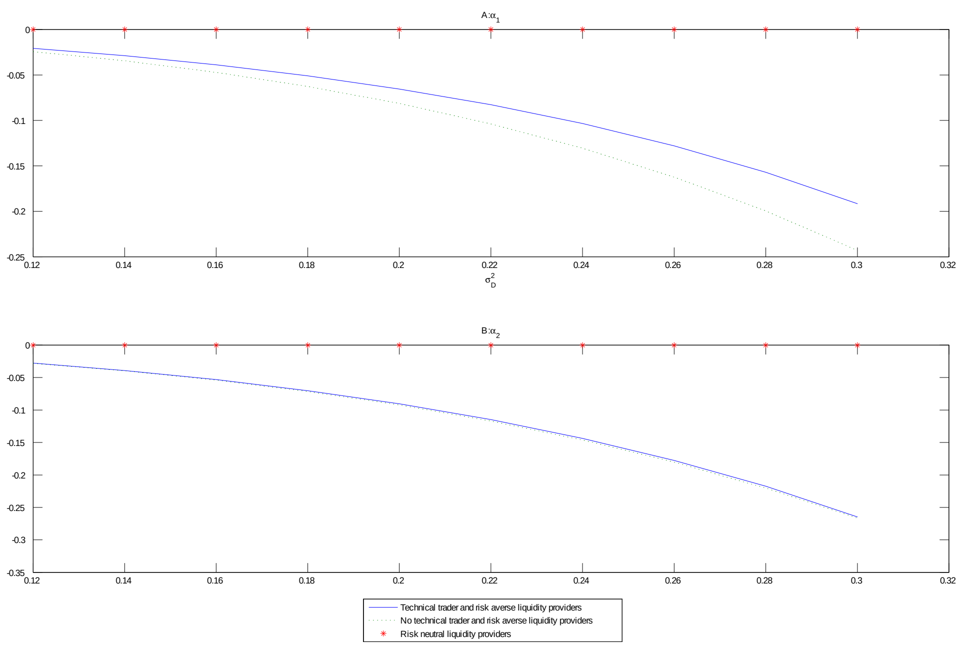

Figure 7: The effect of fundamental risk σ_D on the coefficients of regression of $\frac{D-P_2}{\sqrt{Var[D-P_2]}}$ on $\frac{P_1-P_0}{\sqrt{Var[P_1-P_0]}}$ and $\frac{P_2-P_1}{\sqrt{Var[P_2-P_1]}}$ ($\sigma_U = 1$, $\gamma = 1$).

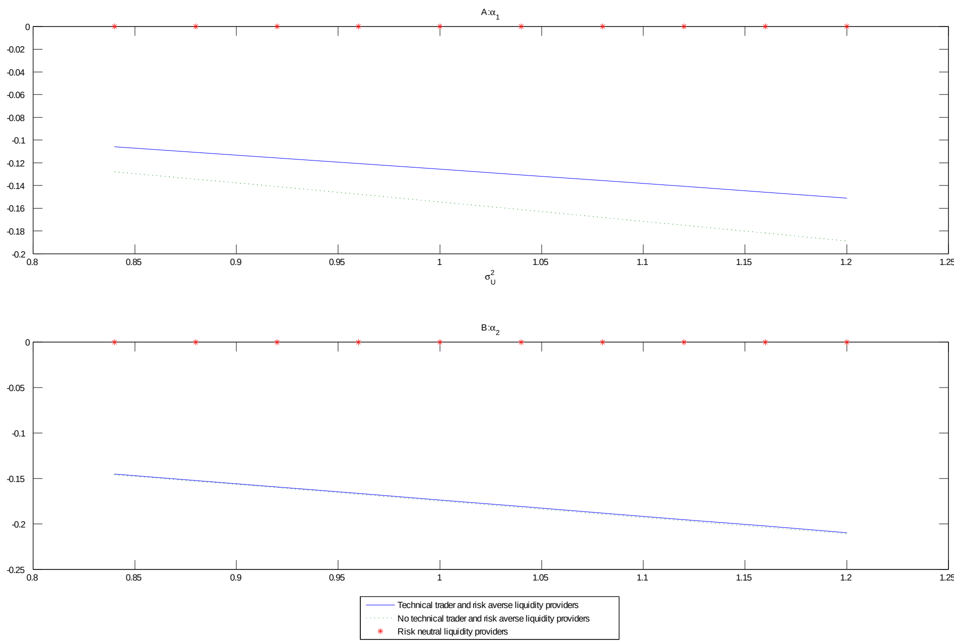

Figure 8: The effect of noise trader risk σ_U on the coefficients of regression of $\frac{D-P_2}{\sqrt{Var[D-P_2]}}$ on $\frac{P_1-P_0}{\sqrt{Var[P_1-P_0]}}$ and $\frac{P_2-P_1}{\sqrt{Var[P_2-P_1]}}$ ($\sigma_D = 0.2$, $\gamma = 1$).

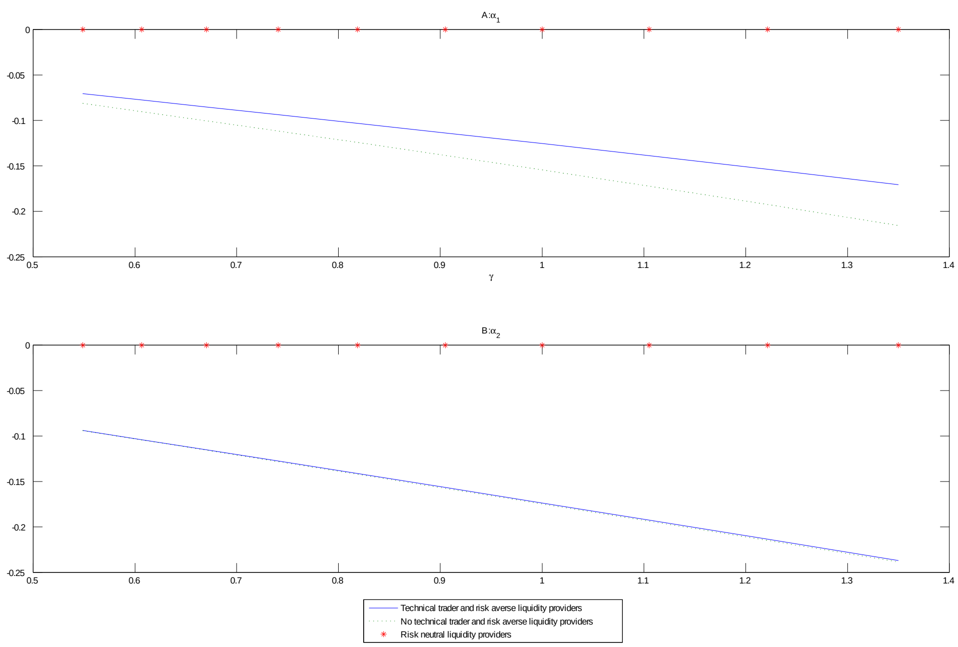

Figure 9: The effect of risk aversion γ on the coefficients of regression of $\frac{D-P_2}{\sqrt{Var[D-P_2]}}$ on $\frac{P_1-P_0}{\sqrt{Var[P_1-P_0]}}$ and $\frac{P_2-P_1}{\sqrt{Var[P_2-P_1]}}$ ($\sigma_D = 0.2, \sigma_U = 1$).

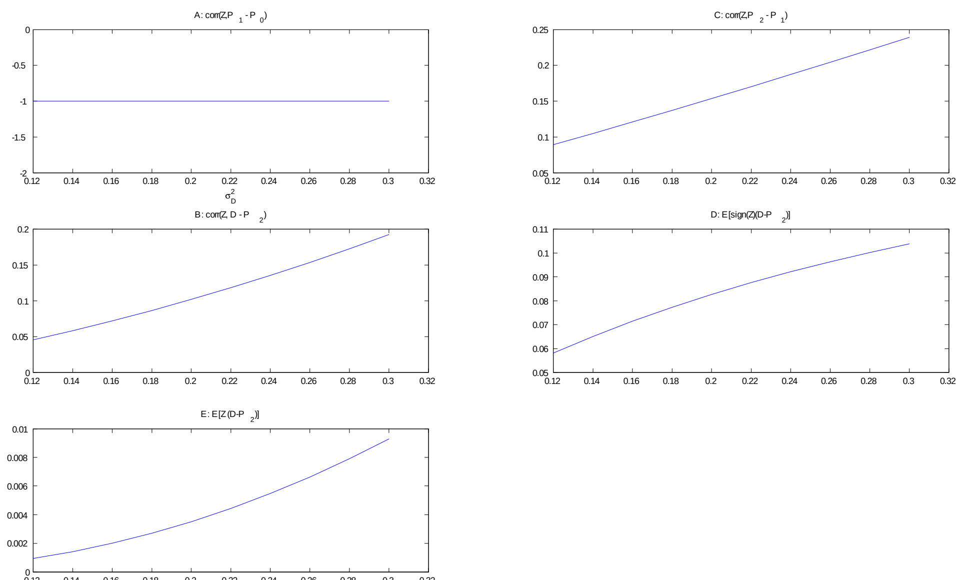

Figure 10: The effects of fundamental risk σ_D on technical trader's trading patterns and expected profits ($\sigma_U = 1$, $\gamma = 1$).

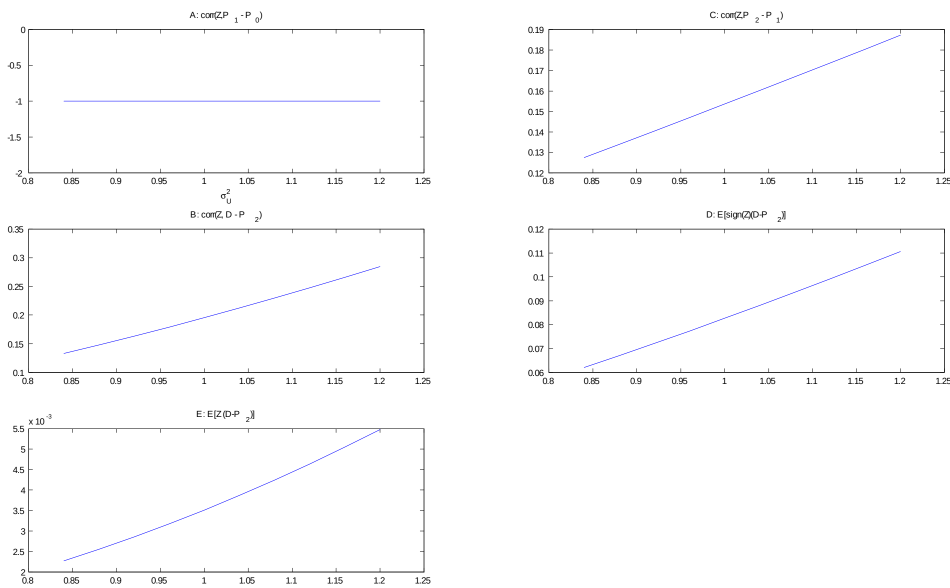

Figure 11: The effects of noise risk σ_U on technical trader's trading patterns and expected profits ($\sigma_D = 0.2$, $\gamma = 1$).

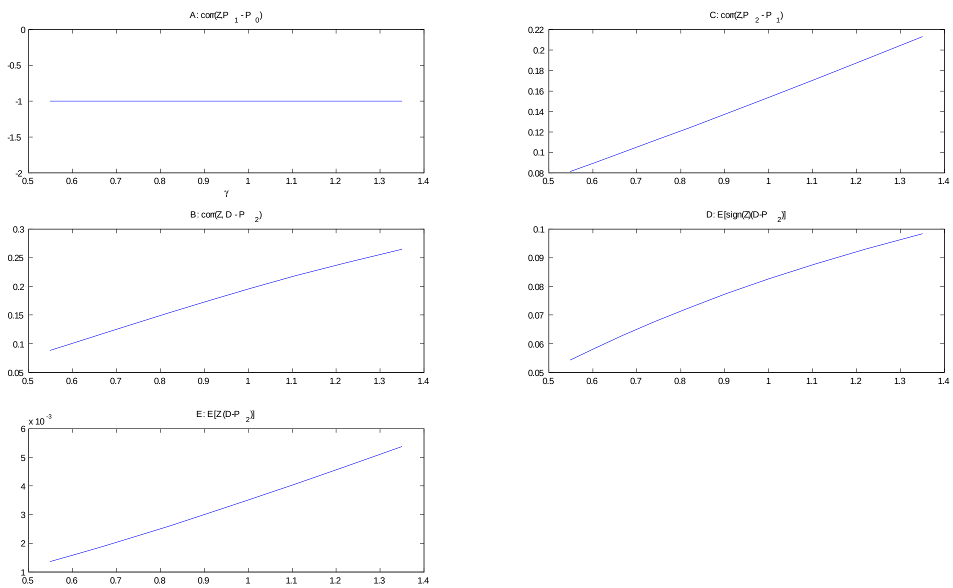

Figure 12: The effects of risk aversion γ on technical trader's trading patterns and expected profits ($\sigma_D = 1, \sigma_U = 1$).