

Social Networks, Information Acquisition, and Asset Prices

Abstract

We analyze a rational expectations equilibrium model to explore the implications of information networks for financial market outcomes. Holding fixed the amount of information in the economy, network communications improve market efficiency, reduce the cost of capital, increase liquidity and trading volume. However, in equilibrium, social communications reduce information acquisition and thus have a negative effect on market efficiency, liquidity, and asset prices. We examine when this negative effect dominates, and derive new testable empirical predictions concerning the relationship between network connectedness and financial market outcomes.

Keywords: Social Communications, Price Informativeness, Information Acquisition, Asset Prices, Liquidity, Volume

JEL Classifications: G14, G12, G11, D82

1 Introduction

A central topic in the studies of financial markets is how much information will be acquired about stock fundamentals and how much the stock price will reflect investors' diverse information. Starting with Grossman and Stiglitz (1980), a theoretical literature has used rational expectations models to study the incentives to acquire costly information on the value of a stock in financial markets.

A more recent empirical literature has separately investigated how information disseminates through agents in financial markets. Several papers document evidence that information sharing with peers via social networks plays an important role for investment decision making such as stock market participation and portfolio choices, for both fund managers and households (see, e.g., Hong, Kubik and Stein (2004, 2005), Ivkovic and Weisbenner (2007), and Cohen, Frazzini and Malloy (2008, 2010)).¹ These studies identify information transfer via social interactions, word-of-mouth communication among friends and neighbors, and shared education networks, etc. Gray and Kern (2011) document that social networks play a direct role in facilitating the price discovery process.² Motivated by these empirical evidence, researchers have started to examine how market information efficiency depends on the structure of a social network (e.g., Colla and Mele (2010) and Ozsoylev and Walden (2010)), but in a setting where information is *exogenously* given.³

This paper combines the two literatures above. We analyze a rational expectations equilibrium model of a competitive market in which heterogeneous traders can learn about a risky asset's payoff from three sources: market price, costly information acquisition, and

¹Social networks can be described as a map of specified ties, such as friendship, between the nodes (individuals) being studied. The nodes to which an individual is connected are the social contacts of that individual.

²In addition to investment decisions and asset prices, network theories have been applied to understand other questions in financial market settings. For example, Allen and Babus (2009) survey applications of social networks for systematic risk, corporate governance, and distribution of primary issues of securities. More generally, social networks have important consequences for a number of economic outcomes, including collaboration among firms, success in job search, educational attainment and participation in crime. Easley and Kleinberg (2010) and Jackson (2008, 2010) provide extensive surveys of the diverse literature on social networks in economics.

³Ozsoylev and Walden (2010) study general forms of network structures and the conditions under which linear rational expectations equilibria exist. Colla and Mele (2010) examine the asset pricing implications of a specific network structure, the cyclical network. They focus on trade correlations among investors of the same network and across investors from different networks.

communications via social network. The network structure is taken to be exogenous.⁴ When a trader decides whether to incur a cost to acquire information, he takes into consideration the expected learning via social communications. In equilibrium, information acquisition and asset prices are determined simultaneously. Traders' conjectures about how much information is revealed through price are fulfilled by their own acquisition activities.

We have three goals. First, we study information acquisition in a financial market with social networks. Second, we examine how network connectedness affects equilibrium market outcomes such as price informativeness, cost of capital, liquidity, and trading volume. Third, we investigate the interactions between information acquisition and communication of information via social networks by comparing the implications of network connectedness for market outcomes when information is endogenously acquired with a cost versus when information is exogenously given.

We need to overcome two main technical difficulties. First, even without social networks, rational expectations equilibrium with endogenous information acquisition generally does not admit closed-form solutions except in some special cases (see, e.g., Grossman and Stiglitz (1980), Verrecchia (1982) and Barlevy and Veronesi (2000)). The difficulty arises from a fixed point problem that needs to be solved in equilibrium.⁵ Second, even when information is exogenous, a linear rational expectations equilibrium may not always exist in the presence of social networks (see Ozsoylev and Walden (2010)).

We extend the CARA-normal setup in a large economy (e.g., Hellwig (1980) and Verrecchia (1982)) with a tractable and realistic social network structure. Our model admits a unique linear rational expectations equilibrium, which we derive analytically. This allows us to do comparative analysis and generate testable empirical predictions about the impact of social networks on asset prices.

We show that how a social network affects market outcomes depends crucially on whether

⁴We acknowledge a large literature developing in social economics on the formation of networks (see e.g., Jackson (2010)). However, as emphasized by Cohen, Frazzini and Malloy (2008), social networks are often formed *ex ante*, and their formation is frequently independent of the information to be transferred.

⁵On one hand, individuals who do not acquire their own signals about a stock's payoff can infer some useful information from stock prices they observe. Thus, their equilibrium demand will depend on the distribution of equilibrium prices. On the other hand, since prices must equate supply and demand in the market, the distribution of equilibrium prices also depends on the demand of individuals who are not fully informed.

private information is exogenously given or endogenously acquired at a cost. Empirical researchers generally treat private information as exogenous, but theoretical researchers tend to treat private information as endogenous. By considering both cases, we isolate the circumstances in which the information acquisition assumption leads to different implications for empirical tests.

When information is exogenous in our model, social communications have two positive effects on reducing the risk of the stock. The first effect is straightforward; information sharing enlarges everyone's information set and thus the precision of a stock's payoff conditional on each trader's information set is higher. Second, sharing information among more friends causes more information to be impounded into the price, thereby improving informational efficiency. This improved market efficiency further reduces the risk of the stock for everyone. As a consequence, when information is exogenous, more social communications increase the average trading aggressiveness of traders, which, in turn, lowers cost of capital, increases stock liquidity and trading volume.

When information is endogenous, in addition to the two positive effects above, social communications have a negative effect on information production arising from the incentive to free ride: In anticipation of learning from informed friends and more informative market price, traders would have less incentive to incur a cost and acquire information on their own, thereby reducing the total amount of information produced in the economy.

Whether the positive or negative effects of social communications dominate depends on the cost of acquiring information. When the cost of acquiring information is sufficiently low, most traders choose to collect information even in the presence of social communications, so that the positive effects dominate. As the information cost becomes larger, fewer people choose to acquire information at a cost when they are connected to more friends, which lowers traders' trading aggressiveness, raises cost of capital, and harms liquidity and volume. The negative effect of social communications eventually overwhelms the positive effects.

A new testable prediction of our model is that the relation between network connectedness and the economic outcome variables such as the cost of capital depends importantly on the information acquisition cost. When the cost of acquiring information is low, the fraction of informed traders is not sensitive to network connectedness, and our model predicts that the

cost of capital is significantly negatively related to network connectedness. In contrast, when the cost of acquiring information is sufficiently high, more social communications lead to a smaller amount of information production, and in this case, the cost of capital is significantly positively related to network connectedness. Similar predictions apply to the relation between price efficiency and the amount of social communications among investors.

Several recent theoretical studies show that social communications improve market efficiency (e.g., Malinova and Smith (2006), Colla and Mele (2010) and Ozsoylev and Walden (2010)). In all of these studies, information is exogenous. We complement these studies by showing that when information is endogenous, social communications can affect economic outcomes in a way that is opposite to the exogenous information case. In addition, we also derive implications of social network for cost of capital and market liquidity, which are not examined in the previous studies.

Our paper also contributes to a large and still growing strand of literature that studies information acquisition and its implications in financial markets. Many interesting topics have been examined in this literature⁶, but our focus on social communications seems unique. We extend the literature by incorporating social networks into a model of information production and generate new implications for asset prices and trading volume.

The remainder of this paper is organized as follows. Section 2 introduces our model including the social network structure. Section 3 solves the financial equilibrium taking as given a fixed fraction of informed traders, and derives comparative statics about how price informativeness, cost of capital, liquidity and trading volume vary with the amount of social communications. Section 4 repeats the same exercises except information market equilibrium (i.e., the fraction of informed traders) is endogenously determined together with the financial market equilibrium. We also make testable empirical predictions. Section 5 concludes.

⁶Recent contributions to this literature include Barlevy and Veronesi (2000), Dow, Goldstein and Guembel (2010), Garcia and Strobl (2010), Garcia and Vanden (2009), Peress (2004, 2010, 2011), Van Nieuwerburgh and Veldkamp (2010).

2 The Model

The model we use is one of pure exchange. There is one period and two assets in the model: a riskfree asset (bond) and a risky asset (stock). The riskfree asset is assumed to pay off in one unit of the single consumption good and has a constant value of 1. The risky asset has an uncertain payoff at the end of the period, denoted \tilde{v} . Assume $\tilde{v} \sim N(\bar{v}, 1/\rho_v)$ with $\bar{v} > 0$ and $\rho_v > 0$.

There is a $[0, 1]$ continuum of rational traders who have constant-absolute-risk-aversion (CARA) utility with a risk aversion coefficient $\gamma > 0$. They are endowed with only riskfree asset at the beginning of the period. Before trade, they choose whether to spend an amount $c > 0$ to become informed. Let μ be the (endogenous) fraction of informed traders. Informed traders receive diverse private signals regarding the stock payoff. A competitive financial market then opens and traders (informed, uninformed and noise traders) trade. Noise traders supply \tilde{x} units of stock per capita to the market. We assume $\tilde{x} \sim N(\bar{x}, 1/\rho_x)$ with $\bar{x} > 0$ and $\rho_x > 0$.⁷

Our model features an information network. In addition to acquiring information at a cost and learning from price, traders can freely communicate to others that are connected to them in the network. We borrow the islands-connections model in the social network literature. There are a total mass of $\frac{1}{N}$ groups (islands) in the economy, each of which has $N \geq 1$ traders. Each member of the group is independently randomly sampled from the total rational traders population. Thus, the fraction of groups having m informed traders and $(N - m)$ uninformed traders is given by the binomial coefficient:

$$\pi_m^N \triangleq \frac{N!}{m!(N-m)!} \mu^m (1-\mu)^{N-m}. \quad (1)$$

Within any group every trader is connected to all other traders in the group, but there are no links across groups.⁸ To ease exposition, we refer to traders in the same group as “friends”.

⁷The assumption of positive supply of the risky asset is important for our analysis on the expected return of risky asset (cost of capital). If $\bar{x} = 0$, then the risk premium of the risky asset is zero in our model because there is no aggregate risk to be borne in this case.

⁸For more details and an equilibrium justification of the network structure we use, see Jackson (2008). Briefly, if the cost of forming a connection within the group is sufficiently lower than across groups, the equilibrium network structure will be that traders are connected to each other within a group and that

In the language of network literature, each group forms a disjoint “complete component” and the “degree” of every agent, i.e., the number of connections each one has, is the same so that the network is “regular”. Our N is equivalent to Ozsoylev and Walden’s (2010) concept of “network connectedness” (β in their model) and Colla and Mele’s (2010) concept of “information linkages” (G in their model). We follow Ozsoylev and Walden (2010) and refer to N as *network connectedness*.

The information structure is as follows. An informed trader i in a group g (after paying a cost c) observes a signal

$$\tilde{s}_{i,g} = \tilde{v} + \tilde{\varepsilon}_{i,g}, \text{ with } \tilde{\varepsilon}_{i,g} \sim N(0, 1/\rho_\varepsilon) \text{ and } \rho_\varepsilon > 0. \quad (2)$$

Informed traders share their signals with other members of the same group. To capture the fact that some private information may be difficult to share (e.g., due to communication barriers), we assume that other traders in group g receive a noisy signal from trader i as follows:

$$\tilde{y}_{i,g} = \tilde{s}_{i,g} + \tilde{\eta}_{i,g}, \text{ with } \tilde{\eta}_{i,g} \sim N(0, 1/\rho_\eta) \text{ and } \rho_\eta \geq 0. \quad (3)$$

We assume that $(\tilde{v}, \{\tilde{\varepsilon}_{i,g}, \tilde{\eta}_{i,g}\}_{i,g}, \tilde{x})$ are mutually independent.

Define

$$\rho_y \triangleq (\rho_\varepsilon^{-1} + \rho_\eta^{-1})^{-1} \in [0, \rho_\varepsilon), \quad (4)$$

which is the precision of signal $\tilde{y}_{i,g}$ conditional on the realization of \tilde{v} . When $\rho_\eta = 0$ (it is impossible to share private information with friends) or when $N = 1$ (there is only one member in each group), social networks do not play a role, reducing our model to a standard Grossman-Stiglitz setup with diverse signals as in Hellwig (1980). We assume throughout the paper that $\rho_\eta > 0$ and $N > 1$.

The timeline is summarized in Figure 1. At the very beginning of the period, traders receive their endowments and groups are formed; rational traders choose whether or not to

groups are disjoint. This network formation process can be easily incorporated into our model, as the relationship choice decision is independent of the trading decision. Our social network can also be viewed as a simultaneous version of Banerjee and Fudenberg’s (2004) “word-of-mouth” model in which each trader trades sequentially and observes the actions of randomly selected N traders. Carlin and Manso (2010) also use a similar setup to study the evolution of investor sophistication.

acquire a private signal at cost c ; informed traders receive their own signals and each trader observes those signals reported by their friends; stock market opens and trading occurs; at the end of the period, stock payoffs are received and all traders consume.

INSERT FIGURE 1 HERE

From the literature on noisy rational expectations equilibria, we know that tractable solutions can often be found in large economies (e.g., Hellwig (1980)). Our model deliberately assumes a *continuum* of traders in the whole economy and a *finite* number of traders in each group to capture the fact that the number of friends (social connections) each trader has is much smaller compared to the number of market participants. More importantly, this assumption implies that no traders have price impacts in our economy so our model avoids the “non-truthful” reporting problem (as pointed out by Ozsoylev and Walden (2010)) and the “schizophrenia” problem (as pointed out by Hellwig (1980)). In addition, the assumption ensures the existence of analytical solutions for any network connectedness N in our economy, as the noises contained in the private signals of informed traders cancel out in the price function.

In the following two sections, we characterize the equilibrium and analyze the implications of social communications. Our focus is on how market efficiency, cost of capital, liquidity and volume depend on network connectedness N . The equilibrium concept that we use is the rational expectations equilibrium, as in Grossman and Stiglitz (1980). In the financial market, traders maximize their expected utility conditioning on the information they acquired (if any), the information communicated by friends, and the market-clearing stock price. In the information market, traders decide whether to pay a cost to become informed to maximize their expected utility generated from their future trading in financial markets. In Section 3, we start by analyzing trading behavior and prices in the financial market, taking the fraction of informed traders μ as given. In Section 4, we determine the equilibrium fraction of informed traders μ^* .

3 Financial Market Equilibrium: Exogenous μ

3.1 Equilibrium Characterization

This section solves the equilibrium stock price, taking as given the fraction μ of informed traders. Suppose traders conjecture the price function

$$\tilde{p} = \alpha_0 + \alpha_v \tilde{v} - \alpha_x \tilde{x}. \quad (5)$$

In this price function α_x measures the effect of noise trading on prices. It captures the market depth. More liquid markets have a smaller α_x . So α_x characterizes market illiquidity. Hence, we use $\frac{1}{\alpha_x}$ to measure stock liquidity.

The information contained in the price is equivalent to the following signal:

$$\tilde{\theta} = \frac{\tilde{p} - \alpha_0 + \alpha_x \bar{x}}{\alpha_v} = \tilde{v} - \frac{\alpha_x}{\alpha_v} (\tilde{x} - \bar{x}), \quad (6)$$

which, conditional on \tilde{v} , is normally distributed with mean \tilde{v} and precision ρ_θ given by

$$\rho_\theta = (\alpha_v/\alpha_x)^2 \rho_x. \quad (7)$$

The endogenous parameter ρ_θ , or equivalently the ratio of $\frac{\alpha_v}{\alpha_x}$, reflects the *price-informativeness* about the fundamental \tilde{v} . Both are measures of *market efficiency*. To see this, note that the literature (e.g., Ozsoylev and Walden (2010) and Peress (2010)) measures market efficiency by the precision of risky asset payoff conditional on its price, i.e., by $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$. By equations (6) and (7), applying Bayes' rule delivers

$$\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \rho_v + \rho_\theta.$$

Since we will fix ρ_v and ρ_x in our subsequent analysis, we can measure market efficiency by ρ_θ , or equivalently by $\frac{\alpha_x}{\alpha_v}$. We use the terms “price-informativeness” and “market efficiency” interchangeably.

We now derive the demand functions of rational traders. Consider a typical group g ,

which has I_g informed traders and $(N - I_g)$ uninformed traders. The information set of an informed trader i is $\mathcal{F}_{i,g}^{IN} = \{\tilde{p}; \tilde{y}_{1,g}, \dots, \tilde{y}_{i-1,g}, \tilde{s}_{i,g}, \tilde{y}_{i+1,g}, \dots, \tilde{y}_{I_g,g}\}$. By the CARA-normal setup, his demand function for the stock is:

$$d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) = \frac{E(\tilde{v} | \mathcal{F}_{i,g}^{IN}) - \tilde{p}}{\gamma \text{Var}(\tilde{v} | \mathcal{F}_{i,g}^{IN})}.$$

Standard calculations show that

$$E(\tilde{v} | \mathcal{F}_{i,g}^{IN}) = \frac{\rho_v \bar{v} + \rho_\theta \tilde{\theta} + \rho_\varepsilon \tilde{s}_{i,g} + \rho_y \sum_{i'=1, i' \neq i}^{I_g} \tilde{y}_{i',g}}{\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y}, \quad (8)$$

$$\text{Var}(\tilde{v} | \mathcal{F}_{i,g}^{IN}) = \frac{1}{\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y}. \quad (9)$$

His demand function is thus

$$d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) = \frac{\rho_v \bar{v} + \rho_\theta \tilde{\theta} + \rho_\varepsilon \tilde{s}_{i,g} + \rho_y \sum_{i'=1, i' \neq i}^{I_g} \tilde{y}_{i',g} - [\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y] \tilde{p}}{\gamma}. \quad (10)$$

The information set of an uninformed trader is $\mathcal{F}_g^{UN} = \{\tilde{p}; \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g}\}$ and his stock demand is

$$d(\tilde{p}; \mathcal{F}_g^{UN}) = \frac{E(\tilde{v} | \mathcal{F}_g^{UN}) - \tilde{p}}{\gamma \text{Var}(\tilde{v} | \mathcal{F}_g^{UN})},$$

where

$$E(\tilde{v} | \mathcal{F}_g^{UN}) = \frac{\rho_v \bar{v} + \rho_\theta \tilde{\theta} + \rho_y \sum_{i=1}^{I_g} \tilde{y}_{i,g}}{\rho_v + \rho_\theta + I_g \rho_y}, \quad (11)$$

$$\text{Var}(\tilde{v} | \mathcal{F}_g^{UN}) = \frac{1}{\rho_v + \rho_\theta + I_g \rho_y}. \quad (12)$$

Thus, an uninformed trader's demand function for stock is

$$d(\tilde{p}; \mathcal{F}_g^{UN}) = \frac{\rho_v \bar{v} + \rho_\theta \tilde{\theta} + \rho_y \sum_{i=1}^{I_g} \tilde{y}_{i,g} - (\rho_v + \rho_\theta + I_g \rho_y) \tilde{p}}{\gamma}. \quad (13)$$

The uninformed and informed demand functions can be unified by the following equation:

$$d(\tilde{p}; \mathcal{F}_{i,g}) = \frac{1}{\gamma} \begin{bmatrix} \rho_v \bar{v} + \rho_\theta \tilde{\theta} + \rho_y \sum_{i=1}^{I_g} \tilde{y}_{i,g} + 1_{\{i=IN\}} (\rho_\varepsilon \tilde{s}_i - \rho_y \tilde{y}_{i,g}) \\ - [\rho_v + \rho_\theta + I_g \rho_y + 1_{\{i=IN\}} (\rho_\varepsilon - \rho_y)] \tilde{p} \end{bmatrix}, \quad (14)$$

where $\mathcal{F}_{i,g}$ is the information set of trader i ($= 1, \dots, N$) in group g and where $1_{\{i=IN\}}$ is equal to 1 if trader i is informed and equal to 0 otherwise.

The aggregate demand function of the group is

$$D(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}) \triangleq \sum_{i=1}^N d(\tilde{p}; \mathcal{F}_{i,g}) = \sum_{i=1}^{I_g} d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) + (N - I_g) d(\tilde{p}; \mathcal{F}_g^{UN}). \quad (15)$$

The market clearing condition is

$$\int_0^{1/N} D(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}) dg = \tilde{x}, \quad (16)$$

where the left-hand side aggregates group demands over the total $\frac{1}{N}$ mass of groups, and the right-hand side is the per capita supply of stock.

Substituting the demand functions (given by equations (10), (13) and (15)) and the expressions for $\tilde{\theta}$ (given by equation (6)) into the market clearing condition (equation (16)), noting that the distribution of groups is given by the binomial distribution characterized by equation (1) and solving for \tilde{p} , and checking the conjectured form of the price function yields the following proposition, whose proof is delegated to Appendix A.

Proposition 1 *There exists a partially revealing rational expectations equilibrium in the financial market, with price function*

$$\tilde{p} = \alpha_0 + \alpha_v \tilde{v} - \alpha_x \tilde{x},$$

where

$$\begin{aligned}\alpha_0 &= \frac{\rho_v \bar{v} + (\alpha_v/\alpha_x) \rho_x \bar{x}}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N-1) \rho_y}, \\ \alpha_v &= \frac{\rho_\theta + \mu \rho_\varepsilon + \mu (N-1) \rho_y}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N-1) \rho_y}, \\ \alpha_x &= \frac{(\alpha_v/\alpha_x) \rho_x + \gamma}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N-1) \rho_y},\end{aligned}$$

where

$$(\alpha_v/\alpha_x) = \frac{\mu \rho_\varepsilon + \mu (N-1) \rho_y}{\gamma} \quad \text{and} \quad \rho_\theta = \frac{\mu^2 [\rho_\varepsilon + (N-1) \rho_y]^2 \rho_x}{\gamma^2}.$$

In particular, equation (32) in Appendix A shows that the price-informativeness measure $\frac{\alpha_v}{\alpha_x}$ is given by

$$\frac{\alpha_v}{\alpha_x} = \frac{1}{\gamma} \left(\sum_{m=0}^N \pi_m^N m \right) \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right).$$

This expression is intuitive. The term of $\left(\sum_{m=0}^N \pi_m^N m \right)$ is the average number of informed traders (and hence signals) in a typical group; the term of $\left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right)$ is the average precision of each signal across N traders in a group (one trader observes the signal perfectly and the remaining traders observe it garbled with noise); and their joint effects determine how much information is incorporated into prices. The expression of $\frac{\alpha_v}{\alpha_x}$ in Proposition 1 is obtained by substituting $\left(\sum_{m=0}^N \pi_m^N m \right) = N\mu$, which is a property of binomial distributions.

3.2 Implications

We now explore the implications of social communications for aggregate outcomes in economies with an exogenous information structure (a fixed μ) by examining how varying N changes market efficiency, cost of capital, liquidity and trading volume. The role of N in determining market outcomes can be well understood by looking at its impact on market efficiency, because its impact on other aggregate variables is closely linked to its impact on market efficiency.

By Proposition 1, increasing the networks connectedness N has a positive effect on market efficiency ρ_θ in a setting with exogenous information, since $\frac{\partial \rho_\theta}{\partial N} = \frac{2\mu^2 [\rho_\varepsilon + (N-1)\rho_y] \rho_x \rho_y}{\gamma^2} > 0$ for a fixed $\mu > 0$. This result is emphasized in the existing literature (e.g., Malinova and Smith

(2006), Colla and Mele (2010) and Ozsoylev and Walden (2010)) and its intuition is quite obvious: Sharing information among more friends causes more information to be impounded into the price, thereby improving informational efficiency.

The positive effect of social communications on market efficiency in turn has implications for the cost of capital, liquidity and trading volume. Following Easley and O'Hara (2004), we define the cost of capital (or expected return from holding the risky asset) as

$$E(\tilde{v} - \tilde{p}) = \frac{\gamma \bar{x}}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu(N-1)\rho_y}, \quad (17)$$

where the equality follows from Proposition 1. The denominator in the definition of cost of capital is the average conditional precision of stock payoff \tilde{v} across all traders: ρ_v is the prior precision, ρ_θ is the precision gleaned from prices, $\mu \rho_\varepsilon$ is the precision coming from the signals directly observed by informed traders, and $\mu(N-1)\rho_y$ is the precision resulting from communicating with informed friends ($\mu(N-1)$ is the average number of informed traders that a typical trader expects to meet and ρ_y is the precision of the signal communicated by an informed friend). Thus, $\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu(N-1)\rho_y$ measures (inversely) traders' average uncertainty regarding firm value.

For a given $\mu > 0$, increasing N will lower the cost of capital, since $\frac{\partial(\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu(N-1)\rho_y)}{\partial N} = \frac{\partial \rho_\theta}{\partial N} + \mu \rho_y > 0$. The intuition is straightforward. Social communications influence stock prices through two channels. First, more information sharing via social networks makes everyone better informed and less uncertain about the stock payoff. This increases the aggregate demand for the stock. Thus, the equilibrium price is higher and the cost of capital is lower. This effect is captured by the term $\mu(N-1)\rho_y$ in the denominator of equation (17). Second, there is an indirect effect on the cost of capital through the revelation of information by the stock price: More information sharing via social networks causes the price system to reveal more information to all traders, which makes the stock less risky and further reduces the cost of capital. This second effect is captured by the term ρ_θ in the denominator of equation (17). These two effects combine to make social communications reduce the cost of capital in an economy with exogenous information.

The illiquidity of the market is measured by α_x , which captures the market depth, namely,

how much a unit of non-information-driven stock transaction can move prices (i.e., by the price equation (5), we have $\frac{\partial \tilde{p}}{\partial \tilde{x}} = -\alpha_x$). Therefore, we can use $\frac{1}{\alpha_x}$ as a liquidity measure: More liquid markets have a larger $\frac{1}{\alpha_x}$. By Proposition 1,

$$\frac{1}{\alpha_x} = \frac{\rho_v + \rho_x (\alpha_v/\alpha_x)^2 + \gamma (\alpha_v/\alpha_x)}{\rho_x (\alpha_v/\alpha_x) + \gamma}. \quad (18)$$

Thus, the effect of N on $\frac{1}{\alpha_x}$ works through its effect on the price-informativeness measure $\frac{\alpha_v}{\alpha_x}$, or equivalently on ρ_θ , since $\rho_\theta = (\alpha_v/\alpha_x)^2 \rho_x$.

An increase of $\frac{\alpha_v}{\alpha_x}$ has two effects on liquidity measure $\frac{1}{\alpha_x}$. First, it increases the average trading aggressiveness of rational traders (i.e., it increases the numerator in equation (18): $\rho_v + \rho_x (\alpha_v/\alpha_x)^2 + \gamma (\alpha_v/\alpha_x)$), because each trader can learn more information from prices. This causes the aggregate demand of rational traders to be more sensitive to prices, and as a result, an extra unit of liquidity supply (i.e., an increase in \tilde{x}) tends to move equilibrium prices by a smaller amount, leading to a higher liquidity.

Second, the improved price-informativeness also has an adverse-selection effect. Suppose there is an extra unit of liquidity supply, which causes the stock price to decline. Seeing the decrease in price, rational traders update downwards their expectations of the stock's fundamental and scale back their share demand, because they can not perfectly disentangle information-based trading from liquidity-based trading and attribute the price decline partly to adverse information regarding stock fundamental. When rational traders rely more on prices to extract information, their demand will drop more, which in turn causes the equilibrium price, in reacting to the initial extra unit of liquidity supply, to decrease by a greater amount, thereby resulting in a larger price impact and a lower liquidity. This effect is reflected in the denominator of equation (18): $\rho_x (\alpha_v/\alpha_x) + \gamma$. Whether an increase in $\frac{\alpha_v}{\alpha_x}$ will improve liquidity depends on the relative strength of these two effects.

Direct computation shows that

$$\frac{\partial (1/\alpha_x)}{\partial (\alpha_v/\alpha_x)} = \frac{[\rho_x (\alpha_v/\alpha_x) + \gamma]^2 - \rho_x \rho_v}{[\rho_x (\alpha_v/\alpha_x) + \gamma]^2}. \quad (19)$$

Thus, $\frac{\partial(1/\alpha_x)}{\partial(\alpha_v/\alpha_x)} > 0$ if and only if

$$\frac{\alpha_v}{\alpha_x} > \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}. \quad (20)$$

Since by Proposition 1, $\frac{\partial(\alpha_v/\alpha_x)}{\partial N} = \frac{\mu\rho_y}{\gamma} > 0$ for a fixed $\mu > 0$ (social communications increase price-informativeness $\frac{\alpha_v}{\alpha_x}$ in economies with exogenous information), then $\frac{\partial(1/\alpha_x)}{\partial N} = \frac{\partial(1/\alpha_x)}{\partial(\alpha_v/\alpha_x)} \frac{\mu\rho_y}{\gamma} > 0$ if and only if condition (20) is satisfied, or equivalently, if and only if the following condition is satisfied:

$$\frac{\mu [\rho_\varepsilon + (N-1)\rho_y]}{\gamma} > \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}, \quad (21)$$

which is obtained by plugging $\frac{\alpha_v}{\alpha_x} = \frac{\mu[\rho_\varepsilon + (N-1)\rho_y]}{\gamma}$ in condition (20).

The effect of social communications on market efficiency, cost of capital and liquidity in an economy with a fixed fraction of informed traders is summarized in the following proposition.

Proposition 2 *When the information structure is exogenous, increasing the connectedness of networks will improve market efficiency and lower cost of capital. That is, for a fixed $\mu > 0$, $\frac{\partial\rho_\theta}{\partial N} > 0$ and $\frac{\partial E(\tilde{v}-\tilde{p})}{\partial N} < 0$. In addition, increasing the connectedness of networks will improve liquidity if and only if condition (21) is satisfied. That is, for a fixed $\mu > 0$, $\frac{\partial(1/\alpha_x)}{\partial N} > 0$ if and only if $\frac{\mu[\rho_\varepsilon + (N-1)\rho_y]}{\gamma} > \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}$.*

Social communications also influence trading volume. The aggregate trading volume is defined as

$$\tilde{q} = \frac{1}{2} \left(\int_0^{1/N} \left[\sum_{i=1}^N |d(\tilde{p}; \mathcal{F}_{i,g})| \right] dg + |\tilde{x}| \right), \quad (22)$$

which is an endogenous function of the underlying random variables (\tilde{v}, \tilde{x}) . Equation (48) in Appendix B gives an analytical expression for \tilde{q} . The complexity of the expression for \tilde{q} precludes simple analytical analysis. Instead we use a numerical example to verify the following intuition: Increasing N will — through the direct information sharing among friends and the improved market efficiency — lower the risk faced by rational traders, so that they will trade stocks more aggressively, which in turn will increase average trading

volume.⁹

INSERT FIGURE 2 HERE

Indeed, Panel (d) of Figure 2 confirms the above intuition with an upward sloping $E(\tilde{q})$ as a function of N . Here, one period is taken to be one year and parameter values are chosen as follows. The expected payoff of the risky asset \bar{v} is normalized to be 1. The *ex ante* payoff precision ρ_v is 25, which gives an annual volatility of about 20%. The risk aversion parameter γ is 2. We normalize the per capita supply of the risky asset to 1, so that $\bar{x} = 1$. The precision of the noise supply is set to 10, that is, $\rho_x = 10$, which corresponds to an annual volatility of liquidity supply equal to about 30% of total supply. We follow Gennotte and Leland (1990) in setting the rational trader's signal-to-noise ratio as 0.2, i.e., $(\rho_\varepsilon/\rho_v) = 0.2$, implying that the precision of the private signal is $\rho_\varepsilon = 5$. We also simply set $\rho_\eta = 5$, meaning that an informed trader can only transfer half of his knowledge to his friends (i.e., $\frac{\rho_y}{\rho_\varepsilon} = \frac{1}{2}$). The exogenous value of μ is set at 0.75.¹⁰ Panels (a)-(c) confirm Proposition 2: An increase in N increases market efficiency and decreases cost of capital, and it also improves liquidity given that condition (21) is satisfied (under the parameter configuration in Figure 2: $\frac{\mu[\rho_\varepsilon+(N-1)\rho_y]}{\gamma} > \frac{\mu\rho_\varepsilon}{\gamma} = 1.88 > 1.38 = \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}$).

4 Information Market Equilibrium: Endogenous μ^*

4.1 Equilibrium Characterization

We now go back to analyze the choice made by rational traders on whether to pay the cost c and become informed. In Appendix C, an argument similar to that of Grossman and Stiglitz (1980) shows that when μ fraction of traders have already purchased the private signal, the

⁹This intuition is also consistent with Proposition 9 (b) of Ozsoylev and Walden (2010) who show that the aggregate trading volume is increasing in network connectedness in markets with low variance of network connectedness, which trivially holds in our economy where each trader has the same connectedness of N .

¹⁰This corresponds to the endogenous equilibrium fraction of informed traders when $N = 1$ and $c = 0.02$ in Section 4.

expected net benefit of the information to a potential purchaser is

$$B(\mu; N) = \frac{\sum_{I_g=0}^{N-1} \left(\pi_{I_g}^{N-1} \frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right)}{\sum_{I_g=0}^{N-1} \left(\pi_{I_g}^{N-1} \frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right)} - e^{\gamma c}, \quad (23)$$

where $\pi_{I_g}^{N-1}$ is the binomial coefficient given by equation (1) and ρ_θ is given by Proposition 1. Here, we explicitly express B as a function of N to emphasize the dependence of learning benefit on the connectedness of networks.

When a trader decides whether to become informed before trade, he will take into account the fact that through social networks, he will meet $I_g = 0, \dots, (N-1)$ informed traders with a probability distribution governed by a binomial distribution. The weighted average expressions of the first term of equation (23) reflects that *ex ante*, the trader does not know how many informed traders he will meet in his network.

The benefit function B determines the equilibrium fraction μ^* of informed traders. If $B(0; N) \leq 0$, i.e., a potential buyer does not benefit from becoming informed when no traders are informed, then $\mu^* = 0$, i.e., an equilibrium exists in the information market when no one purchases the information. If $B(1; N) \geq 0$, a potential buyer is strictly better off by being informed when all other traders are also informed, then it is an information market equilibrium when all traders are informed, i.e., $\mu^* = 1$. Given an interior fraction of informed traders ($0 < \mu^* < 1$), if every potential buyer is indifferent to becoming informed versus remaining uninformed, i.e., $B(\mu^*; N) = 0$, then that fraction μ^* is an information market equilibrium.

Examining the expression of learning benefit in equation (23), we find that the numerator and the denominator of the first term are taking expectations with respect to a binomial distribution, that is,

$$B(\mu; N) = \frac{E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right)}{E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right)} - e^{\gamma c},$$

where the operator of $E_{I_g}^{N-1}(\cdot)$ means taking expectation with the random variable I_g , which

follows a binomial distribution with parameters $(N - 1)$ and μ . It is rather difficult to work directly with $E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right)$ and $E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right)$. We therefore take the following first-order approximation:

$$\begin{aligned} E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right) &\approx \frac{1}{\sqrt{\rho_v + \rho_\theta + E_{I_g}^{N-1}(I_g) \rho_y}}, \\ E_{I_g}^{N-1} \left(\frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right) &\approx \frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + E_{I_g}^{N-1}(I_g) \rho_y}}. \end{aligned}$$

So, we have

$$B(\mu; N) \approx \hat{B}(\mu; N) \triangleq \frac{\frac{1}{\sqrt{\rho_v + \rho_\theta + E_{I_g}^{N-1}(I_g) \rho_y}}}{\frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + E_{I_g}^{N-1}(I_g) \rho_y}}} - e^{\gamma c} = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \rho_\theta + (N - 1) \mu \rho_y}} - e^{\gamma c}, \quad (24)$$

where the last equality follows from $E_{I_g}^{N-1}(I_g) = (N - 1) \mu$, which is the average number of informed traders that a trader expects to meet *ex ante*. In the following analysis, we work with the above approximation function $\hat{B}(\mu; N)$. We also use numerical analysis to verify the validity of all the propositions below.

The expression of $\hat{B}(\mu; N)$, i.e.,

$$\hat{B}(\mu; N) = \sqrt{1 + \frac{\rho_\varepsilon}{\underbrace{\rho_v + \rho_\theta}_{\text{learn from prices}} + \underbrace{(N - 1) \mu \rho_y}_{\text{learn from friends}}}} - e^{\gamma c}, \quad (25)$$

describes the impact of social communications on the learning benefit. Two effects are at work here. First, direct learning from friends will affect the incentive of acquiring information and this channel manifests itself through the term $(N - 1) \mu \rho_y$ in equation (25): $(N - 1) \mu$ is the average number of informed friends and ρ_y is the precision of the signal passed on from an informed friend. Second, since traders also learn from market price, the improved market efficiency will affect the learning incentive, too, and this channel is reflected by the term ρ_θ in equation (25).

There are two important properties of function \hat{B} . First, for a fixed N , function \hat{B}

decreases with the fraction of informed traders μ , since by the expression of ρ_θ in Proposition 1, we have

$$\hat{B}(\mu; N) = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \mu^2 [\rho_\varepsilon + (N-1)\rho_y]^2 \rho_x / \gamma^2 + (N-1)\mu\rho_y}} - e^{\gamma c}. \quad (26)$$

This suggests that the expected gain to be informed is small when there are already many informed traders. This result is a generalized version of the Grossman-Stiglitz effect (1980): Given a social network, as more traders become informed, an uninformed trader expects to glean more information from both friends and prices, which reduces his incentive to become informed, meaning that learning is a substitute. The fact that $\hat{B}(\mu, N)$ is monotonic in μ also implies that there is a unique information market equilibrium $\mu^* \in [0, 1]$.

Second, for a fixed μ , function \hat{B} decreases with network connectedness N , too. Thus, increasing the connectedness N of networks will shift downward the function $\hat{B}(\cdot; N)$, leading to a smaller equilibrium fraction μ^* of informed traders. This result is intuitive: Social communications give traders a chance to learn from each other and also cause prices to reveal more information; as a result, the incentive to become informed is reduced, resulting in a smaller fraction of informed traders.

Figure 3 gives an illustration of the above two results for the original learning benefit function B (not \hat{B}). Here the parameters take the same values as in Figure 2. Panel (a) of Figure 3 displays the learning benefit function $B(\cdot; N)$. Indeed, $B(\cdot; N)$ is downward sloping and increasing N moves downward function $B(\cdot; N)$. The equilibrium fraction μ^* of informed traders is determined by the intersection point at which function $B(\cdot; N)$ crosses zero. Panel (b) of Figure 3 plots μ^* against the connectedness N of networks, which confirms the result that increasing N will decrease μ^* .

INSERT FIGURE 3 HERE

The following proposition characterizes the information market equilibrium and summarizes the effect of social communications on the equilibrium fraction of informed traders.

Proposition 3 *Suppose $\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}} < e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$. Then, for any network connectedness N , there is a unique equilibrium fraction of informed traders $\mu^* \in (0, 1)$ given*

by

$$\mu^* = \frac{\frac{2}{(N-1)\rho_y} \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right)}{1 + \sqrt{1 + 4 \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right) \frac{\rho_x}{\gamma^2} \left(1 + \frac{\rho_\varepsilon}{(N-1)\rho_y} \right)^2}}.$$

Increasing the network connectedness N will reduce the endogenous fraction of informed traders; that is $\frac{d\mu^*}{dN} < 0$.

The proof of Proposition 3 is delegated to Appendix D. The condition of $\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}} < e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$ implies that $\hat{B}(1; 1) < 0 < \hat{B}(0; 1)$, which ensures that the endogenous fraction of informed traders takes a value between 0 and 1. Intuitively, the condition of $\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}} > e^{\gamma c}$ says that when no one is informed, it is beneficial for an uninformed trader to become informed, and the condition of $e^{\gamma c} > \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}}$ says that it is not an equilibrium for everyone to be informed.

In contrast, if c is sufficiently small so that $e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}}$, then for a fixed c satisfying this condition, there exists a positive integer \bar{N} such that for any $N \in [1, \bar{N}]$, we have $\hat{B}(1; N) > 0$ and hence $\mu^* = 1$. Thus, as long as $N \in [1, \bar{N}]$, the relation between the market variables and N in the endogenous information case is the same as that in the exogenous information case (see Section 3.2).

Finally, if the cost c of acquiring information is too high so that $e^{\gamma c} > \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$, then $\hat{B}(\mu; N) \leq \hat{B}(0; N) < 0$, and as a result, no one chooses to acquire information, i.e., $\mu^* = 0$. In this case, social communications does not matter as there is no information to be shared among friends.

4.2 Implications

Throughout this subsection, we assume that the information cost c satisfies the condition given in Proposition 3. We demonstrate that with endogenous information acquisition, the implications of social communications for economic outcomes could be the opposite of those under exogenous information. For example, when information acquisition is endogenous, increasing the connectedness N of networks will — through decreasing the equilibrium fraction μ^* of informed traders — reduce market efficiency and increase cost of capital.

When μ^* is between 0 and 1, it is approximately determined by setting $\hat{B}(\mu^*; N) = 0$, that is,

$$\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \rho_\theta^* + (N-1)\mu^*\rho_y}} = e^{\gamma c}, \quad (27)$$

where $\rho_\theta^* = \frac{\mu^{*2}[\rho_\varepsilon + (N-1)\rho_y]^2 \rho_x}{\gamma^2}$ is the market efficiency measure evaluated at the equilibrium fraction μ^* of informed traders. Equation (27) is equivalent to the following relation:

$$\rho_v + \rho_\theta^* + \mu^*(N-1)\rho_y = \frac{\rho_\varepsilon}{e^{2\gamma c} - 1}. \quad (28)$$

The expression of μ^* in Proposition 3 implies that increasing N will increase the expected precision $\mu^*(N-1)\rho_y$ of signals shared with friends. Then, to maintain equation (28), market efficiency ρ_θ^* has to decrease. Thus, increasing N will reduce market efficiency when information is endogenous.

The reduced market efficiency ρ_θ^* due to social communications in turn has implications for liquidity. Recall that equation (18) indicates that any impact of N on liquidity $\frac{1}{\alpha_x}$ occurs through its impact on the equilibrium price-informativeness measure $\frac{\alpha_v}{\alpha_x}$ (or the measure ρ_θ^*). By equation (20), $\frac{\partial(1/\alpha_x)}{\partial(\alpha_v/\alpha_x)} > 0$ if and only if $\frac{\alpha_v}{\alpha_x} > \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}$. Since in the economy with endogenous information, when the information cost c satisfies the condition given in Proposition 3, increasing N will decrease $\frac{\alpha_v}{\alpha_x}$ (social communications hurt market efficiency), social communications improve liquidity if and only if the following condition is satisfied:

$$\frac{\mu^* [\rho_\varepsilon + (N-1)\rho_y]}{\gamma} < \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}. \quad (29)$$

Note that the above condition is exactly the opposite of the condition in economies with exogenous information (equation (21)).

In economies with endogenous information, the average risk faced by traders increase with network connectedness. Thus more social communications will increase the cost of capital. To see this, recall that the average conditional precision of stock payoff \tilde{v} across all traders (which is the denominator in the expression of the cost of capital, see equation (17)) is:

$$\rho_v + \rho_\theta^* + \mu^*\rho_\varepsilon + \mu^*(N-1)\rho_y.$$

By equation (28), we have

$$\rho_v + \rho_\theta^* + \mu^* \rho_\varepsilon + \mu^* (N - 1) \rho_y = \mu^* \rho_\varepsilon + \frac{\rho_\varepsilon}{e^{2\gamma c} - 1};$$

According to Proposition 3, increasing N will decrease μ^* , which leads to a higher average risk faced by traders, and thus a larger cost of capital.

We summarize the above results in the following proposition, which uses full differentiation operator $\frac{d}{dN}$ to emphasize the fact that we have taken into account of the impact of N on μ^* .

Proposition 4 *Assume $\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}} < e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$. Increasing the connectedness N of social network will harm market efficiency ρ_θ^* and raise the cost of capital; that is, $\frac{d\rho_\theta^*}{dN} < 0$ and $\frac{dE(\tilde{v}-\tilde{p})}{dN} > 0$. Increasing N will reduce liquidity when $\frac{\mu^*[\rho_\varepsilon + (N-1)\rho_y]}{\gamma} > \sqrt{\frac{\rho_v}{\rho_x}} - \frac{\gamma}{\rho_x}$.*

Moreover, in economies with endogenous information, social communications affect trading volume differently from the exogenous information case. Again, we use the same numerical example as in Subsection 3.2 to examine the role of N in determining average trading volume $E(\tilde{q})$. In this case, we expect increasing N will decrease volume, because, once the fraction of informed traders is endogenous, according to Proposition 4, increasing N will cause traders to trade stocks less aggressively. Indeed, Panel (d) of Figure 4 verifies this result. Panels (a)-(c) of Figure 4 are also consistent with the results regarding market efficiency, cost of capital and liquidity, as described by Proposition 4. More importantly, comparing Figures 2 and 4, we find that social communications can have exactly opposite effects on aggregate outcomes in economies with exogenous information versus those with endogenous information.

INSERT FIGURE 4 HERE

4.3 New Empirical Predictions

Proposition 2 shows that when information is exogenous, the cost of capital monotonically decreases with network connectedness. However, our model with endogenous information

structure implies that the relation between cost of capital and network connectedness depends importantly on the information acquisition cost.

We distinguish two cases. First, when the cost of acquiring information is low, the fraction of informed traders is not sensitive to network connectedness, and thus can be regarded as exogenous. In this case, our model generates the same qualitative prediction as the exogenous information case (see Proposition 2): Cost of capital decreases with network connectedness. Second, when the cost of acquiring information is sufficiently high, Proposition 4 shows that cost of capital increases with network connectedness.

Prediction 1. *(a) For firms with low information acquisition cost, the cost of capital is significantly negatively related to network connectedness. (b) For firms with high information acquisition cost, the cost of capital is significantly positively related to network connectedness.*

Hong, Kubik and Stein (2008) find that stocks in regions with low population density tend to have higher prices. They interpret this result as an “only game in town” effect: Regions with low population density are home to relatively few firms per capita; local bias of investors and limited supply of shares drive up prices. We advance an alternative interpretation for their finding that is consistent with our Prediction 1(b): A low population density corresponds to fewer connections possessed by (local) traders on average — a smaller N in our model — and hence a larger amount of private information is produced for firms headquartered in a low population density area, leading to a higher price. Under this explanation, our Prediction 1(b) implies that the result of Hong, Kubik and Stein (2008) should be stronger for firms with relatively high information acquisition cost.

Similarly, our model predicts that the relation between price informativeness or market efficiency and network connectedness depends importantly on the information acquisition cost.¹¹

Prediction 2. *(a) For firms with low information acquisition cost, market efficiency is higher when there are more social communications among their investors. (b) For firms with high information acquisition cost, market efficiency is lower when there are more social communications among their investors.*

¹¹For empirical predictions about the impact of social communications on trading volume, it is better to use dynamic models (see e.g., Colla and Mele (2010), and Hong, Hong and Ungureanu (2011)).

To test the predictions above, one needs to measure, for a given firm, the cost for investors to become informed and the amount of social interactions its investors have. There are several possible measures for the cost for investors to become informed. The first measure is the number of analysts following the firm. More information is available to outsiders about the firm when it is followed by more analysts. The second measure is the dispersion of analyst forecasts. A lack of consensus among analysts suggests it is difficult for outsiders to become informed about the firm. The third measure is the analyst forecast error. Large forecast errors indicate a greater difficulty of becoming informed. Another relevant measure of the cost for investors to be informed is the amount of media coverage. Firms with more media coverage have lower information cost.

The social networks of investors can be identified either directly or indirectly. First, one can rely on datasets on individual households, such as the Finnish Central Securities Depository dataset used by Grinblatt and Keloharju (2000), to directly build a proxy for network connectedness. Second, one can use datasets containing individual trading information, such as the datasets used by Barber *et al.* (2009), to examine the similarities of traders' portfolio holdings and indirectly infer the social network structure. An example in case is Ozsoylev *et al.* (2011) who use an account level dataset of all trades on the Istanbul Stock Exchange to identify traders who are similar in their trading behavior as linked in an empirical investor network (EIN).¹²

5 Concluding Remarks

In this paper, we analyze a rational expectations equilibrium model of private information to study the implications of social communications for financial markets. In our model, traders can acquire relevant information about a risky asset's payoff at some cost. In addition, our model incorporate information sharing via social networks. When the fraction of informed investors is fixed exogenously, social communications improve market efficiency, reduce the cost of capital, and tend to increase liquidity and trading volume. However, social communications also reduce investors' incentive to acquire information and thus have a negative

¹²See also Ozsoylev and Walden (2010) for discussions on the identification of investor social networks.

effect on market efficiency. We examine when this negative effect dominates and makes new testable empirical predictions.

Our model is designed to be simple and parsimonious for tractability. But our results hold more generally. For example, we have assumed that the cost to become privately informed and the precision of the private signal are fixed. More generally, we can allow traders to spend different costs and acquire information with different precisions by specifying the information acquisition cost to be an increasing function of the precision of private information. As long as costly information acquisition and social communications are substitutes, a free-riding problem emerges: Anticipating the exchange of information via social communications, traders are tempted to acquire less information compared to the case without social communications. This in turn leads to a negative effect of social communications on financial market outcome variables such as market efficiency and cost of capital.

Finally, we want to emphasize that social network may harm market efficiency via alternative channels. For example, it might be possible that social networks not only transmit information, but also noise (e.g., Acemoglu, Ozdaglar and ParandehGheibi (2010)). We leave it to future research to further explore the implications of social network for financial market.

Appendix

A. Proof of Proposition 1.

Substituting equations (10), (13) into (15)) delivers the following expression of the group demand function:

$$D\left(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}\right) = \frac{1}{\gamma} \left\{ \begin{array}{l} N\left(\rho_v \bar{v} + \rho_\theta \tilde{\theta}\right) + [\rho_\varepsilon + \rho_y(N-1)] I_g \tilde{v} \\ + [\rho_\varepsilon + \rho_y(N-1)] \sum_{i=1}^{I_g} \tilde{\varepsilon}_{i,g} + \rho_y(N-1) \sum_{i=1}^{I_g} \tilde{\eta}_{i,g} \\ - [N(\rho_v + \rho_\theta) + I_g(\rho_\varepsilon + (N-1)\rho_y)] \tilde{p} \end{array} \right\}. \quad (30)$$

Now consider the aggregate demand of groups which include m informed traders. The total mass of such groups is $\frac{\pi_m^N}{N}$. Thus, by equation (30), their aggregate demand is

$$\begin{aligned} & \int_{\{g:I_g=m\}} D\left(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}\right) dg \\ &= \int_{\{g:I_g=m\}} \frac{1}{\gamma} \left\{ \begin{array}{l} N\left(\rho_v \bar{v} + \rho_\theta \tilde{\theta}\right) + [\rho_\varepsilon + \rho_y(N-1)] m \tilde{v} \\ + [\rho_\varepsilon + \rho_y(N-1)] \sum_{i=1}^m \tilde{\varepsilon}_{i,g} + \rho_y(N-1) \sum_{i=1}^m \tilde{\eta}_{i,g} \\ - [N(\rho_v + \rho_\theta) + m(\rho_\varepsilon + (N-1)\rho_y)] \tilde{p} \end{array} \right\} dg \\ &= \frac{\pi_m^N}{N} \left\{ \begin{array}{l} N\left(\rho_v \bar{v} + \rho_\theta \tilde{\theta}\right) + [\rho_\varepsilon + \rho_y(N-1)] m \tilde{v} \\ - [N(\rho_v + \rho_\theta) + m(\rho_\varepsilon + (N-1)\rho_y)] \tilde{p} \end{array} \right\} \\ & \quad + \int_{\{g:I_g=m\}} \frac{1}{\gamma} \left\{ [\rho_\varepsilon + \rho_y(N-1)] \sum_{i=1}^m \tilde{\varepsilon}_{i,g} + \rho_y(N-1) \sum_{i=1}^m \tilde{\eta}_{i,g} \right\} dg. \end{aligned}$$

The second term of the above equation is 0 by law of large numbers. Thus,

$$\begin{aligned} & \int_{\{g:I_g=m\}} D\left(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}\right) dg \\ &= \frac{\pi_m^N}{\gamma} \left\{ \rho_v \bar{v} + \rho_\theta \tilde{\theta} + \left(\frac{1}{N}\rho_\varepsilon + \frac{N-1}{N}\rho_y\right) m \tilde{v} - \left[\rho_v + \rho_\theta + \left(\frac{1}{N}\rho_\varepsilon + \frac{N-1}{N}\rho_y\right) m\right] \tilde{p} \right\}. \end{aligned}$$

So, the aggregate demand of the whole economy is

$$\begin{aligned} & \int_0^{\frac{1}{N}} D\left(\tilde{p}; \mathcal{F}_{1,g}^{IN}, \dots, \mathcal{F}_{I_g,g}^{IN}, \mathcal{F}_g^{UN}\right) dg \\ &= \sum_{m=0}^N \left[\frac{\pi_m^N}{\gamma} \left\{ \rho_v \bar{v} + \rho_\theta \tilde{\theta} + \left(\frac{1}{N}\rho_\varepsilon + \frac{N-1}{N}\rho_y\right) m \tilde{v} - \left[\rho_v + \rho_\theta + \left(\frac{1}{N}\rho_\varepsilon + \frac{N-1}{N}\rho_y\right) m\right] \tilde{p} \right\} \right]. \end{aligned}$$

Plugging in the above expression into the market clearing condition (equation (16)), we have

$$\begin{aligned} & \left[\rho_v + \rho_\theta + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right) \right] \tilde{p} \\ &= \left(\rho_v \bar{v} + \rho_\theta \tilde{\theta} \right) + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right) \tilde{v} - \gamma \tilde{x}. \end{aligned} \quad (31)$$

By noting that $\tilde{\theta} = \frac{\tilde{p} - \alpha_0 + \alpha_x \bar{x}}{\alpha_v}$ and comparing coefficients with equation (5), we have

$$\frac{\alpha_v}{\alpha_x} = \frac{\left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right)}{\gamma}. \quad (32)$$

Then, plugging in $\tilde{\theta} = \tilde{v} - \frac{\alpha_x}{\alpha_v} (\tilde{x} - \bar{x})$ into equation (31) yields

$$\begin{aligned} \alpha_0 &= \frac{\rho_v \bar{v} + (\alpha_v / \alpha_x) \rho_x \bar{x}}{\rho_v + \rho_\theta + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right)}, \\ \alpha_v &= \frac{\rho_\theta + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right)}{\rho_v + \rho_\theta + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right)}, \\ \alpha_x &= \frac{(\alpha_v / \alpha_x) \rho_x + \gamma}{\rho_v + \rho_\theta + \left(\frac{1}{N} \rho_\varepsilon + \frac{N-1}{N} \rho_y \right) \left(\sum_{m=0}^N \pi_m^N m \right)}. \end{aligned}$$

By equation (1), we have $\sum_{m=0}^N \pi_m^N m = N\mu$. Further simplifying delivers Proposition 1.

B. Analytical Expression of Trading Volume.

By law of large numbers, we know the aggregate trading volume coming from rational traders is

$$\begin{aligned} & \int_0^{1/N} \left[\sum_{i=1}^N |d(\tilde{p}; \mathcal{F}_{i,g})| \right] dg \\ &= \frac{\sum_{I_g=0}^N \pi_{I_g}^N \sum_{i=1}^N E_{(\varepsilon_{i,g}, \eta_{i,g})_{i=1}^{I_g}} (|d(\tilde{p}; \mathcal{F}_{i,g})| | \tilde{v}, \tilde{x})}{N}, \end{aligned} \quad (33)$$

where $E_{(\varepsilon_{i,g}, \eta_{i,g})_{i=1}^{I_g}} (d(\tilde{p}; \mathcal{F}_{i,g}) | \tilde{v}, \tilde{x})$ means taking expectation of $d(\tilde{p}; \mathcal{F}_{i,g})$ with respect to random variables $(\varepsilon_{i,g}, \eta_{i,g})_{i=1}^{I_g}$ conditional on the observation of (\tilde{v}, \tilde{x}) .

Plugging the expressions of $\tilde{\theta}$ (equation (6)) and \tilde{p} (equation (5)) into the demand function

$d(\tilde{p}; \mathcal{F}_{i,g}^{IN})$ of an informed trader in group g (equation (10)), we can rewrite $d(\tilde{p}; \mathcal{F}_{i,g}^{IN})$ as

$$d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) = \frac{1}{\gamma} \left[h_{0,IN}^{I_g} + h_{v,IN}^{I_g} \tilde{v} + h_{x,IN}^{I_g} \tilde{x} + \rho_\varepsilon \tilde{\varepsilon}_{i,g} + \rho_y \sum_{i'=1, i' \neq i}^{I_g} \tilde{\varepsilon}_{i',g} + \rho_y \sum_{i'=1, i' \neq i}^{I_g} \tilde{\eta}_{i',g} \right], \quad (34)$$

where

$$h_{0,IN}^{I_g} = \rho_v \bar{v} - \alpha_0 [\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y] + \rho_\theta \frac{\alpha_x}{\alpha_v} \bar{x}, \quad (35)$$

$$h_{v,IN}^{I_g} = \rho_\theta + \rho_\varepsilon + \rho_y (I_g - 1) - \alpha_v [\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y], \quad (36)$$

$$h_{x,IN}^{I_g} = -\rho_\theta \frac{\alpha_x}{\alpha_v} + \alpha_x [\rho_v + \rho_\theta + \rho_\varepsilon + (I_g - 1) \rho_y]. \quad (37)$$

$d(\tilde{p}; \mathcal{F}_{i,g}^{IN})$ follows a normal distribution given (\tilde{v}, \tilde{x}) and the conditional mean and standard deviation are given by:

$$\mu_{IN}^{I_g}(\tilde{v}, \tilde{x}) \equiv E[d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) | \tilde{v}, \tilde{x}] = \frac{1}{\gamma} \left(h_{0,IN}^{I_g} + h_{v,IN}^{I_g} \tilde{v} + h_{x,IN}^{I_g} \tilde{x} \right), \quad (38)$$

$$\sigma_{IN}^{I_g} \equiv \sqrt{Var[d(\tilde{p}; \mathcal{F}_{i,g}^{IN}) | \tilde{v}, \tilde{x}]} = \frac{1}{\gamma} \sqrt{I_g \rho_y + \rho_\varepsilon - \rho_y}. \quad (39)$$

Conditional on given (\tilde{v}, \tilde{x}) , $|d(\tilde{p}; \mathcal{F}_{i,g}^{IN})|$ follows a folded normal distribution and its conditional mean is thus given by

$$E_{(\varepsilon_{i,g}, \eta_{i,g})_{i=1}^{I_g}} (|d(\tilde{p}; \mathcal{F}_{i,g}^{IN})| | \tilde{v}, \tilde{x}) = \sigma_{IN}^{I_g} \sqrt{\frac{2}{\pi}} e^{-\frac{[\mu_{IN}^{I_g}(\tilde{v}, \tilde{x})]^2}{2(\sigma_{IN}^{I_g})^2}} + \mu_{IN}^{I_g}(\tilde{v}, \tilde{x}) \left[1 - 2\Phi \left(-\frac{\mu_{IN}^{I_g}(\tilde{v}, \tilde{x})}{\sigma_{IN}^{I_g}} \right) \right], \quad (40)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

We can similarly rewrite the demand function of an uninformed trader as

$$d(\tilde{p}; \mathcal{F}_g^{UN}) = \frac{1}{\gamma} \left[h_{0,UN}^{I_g} + h_{v,UN}^{I_g} \tilde{v} + h_{x,UN}^{I_g} \tilde{x} + \rho_y \sum_{i=1}^{I_g} \tilde{\varepsilon}_{i,g} + \rho_y \sum_{i=1}^{I_g} \tilde{\eta}_{i,g} \right], \quad (41)$$

where

$$h_{0,UN}^{I_g} = \rho_v \bar{v} + \rho_\theta \frac{\alpha_x}{\alpha_v} \bar{x} - \alpha_0 (\rho_v + \rho_\theta + I_g \rho_y), \quad (42)$$

$$h_{v,UN}^{I_g} = \rho_\theta + I_g \rho_y + \alpha_v (\rho_v + \rho_\theta + I_g \rho_y), \quad (43)$$

$$h_{x,UN}^{I_g} = -\rho_\theta \frac{\alpha_x}{\alpha_v} + \alpha_x (\rho_v + \rho_\theta + I_g \rho_y). \quad (44)$$

Similarly,

$$E_{(\varepsilon_{i,g}, \eta_{i,g})_{i=1}^{I_g}} (|d(\tilde{p}; \mathcal{F}_g^{UN})| | \tilde{v}, \tilde{x}) = \sigma_{UN}^{I_g} \sqrt{\frac{2}{\pi}} e^{-\frac{[\mu_{UN}^{I_g}(\tilde{v}, \tilde{x})]^2}{2(\sigma_{UN}^{I_g})^2}} + \mu_{UN}^{I_g}(\tilde{v}, \tilde{x}) \left[1 - 2\Phi \left(-\frac{\mu_{UN}^{I_g}(\tilde{v}, \tilde{x})}{\sigma_{UN}^{I_g}} \right) \right], \quad (45)$$

where

$$\mu_{UN}^{I_g} \equiv E [d(\tilde{p}; \mathcal{F}_g^{UN}) | \tilde{v}, \tilde{x}] = \frac{1}{\gamma} \left(h_{0,UN}^{I_g} + h_{v,UN}^{I_g} \tilde{v} + h_{x,UN}^{I_g} \tilde{x} \right), \quad (46)$$

$$\sigma_{UN}^{I_g} \equiv \sqrt{Var [d(\tilde{p}; \mathcal{F}_g^{UN}) | \tilde{v}, \tilde{x}]} = \frac{1}{\gamma} \sqrt{I_g \rho_g}. \quad (47)$$

Plugging equations (40), (45) and (33) into equation (22), the aggregate volume is:

$$\begin{aligned} \tilde{q} &= \frac{1}{2} \left(\int_0^{1/N} \left[\sum_{i=1}^N |d(\tilde{p}; \mathcal{F}_{i,g})| \right] dg + |\tilde{x}| \right) \\ &= \frac{1}{2} \sum_{I_g=0}^N \pi_{I_g}^N \left\{ \begin{array}{l} \frac{I_g}{N} \left[\begin{array}{l} \sigma_{IN}^{I_g} \sqrt{\frac{2}{\pi}} e^{-\frac{[\mu_{IN}^{I_g}(\tilde{v}, \tilde{x})]^2}{2(\sigma_{IN}^{I_g})^2}} \\ + \mu_{IN}^{I_g}(\tilde{v}, \tilde{x}) \left[1 - 2\Phi \left(-\frac{\mu_{IN}^{I_g}(\tilde{v}, \tilde{x})}{\sigma_{IN}^{I_g}} \right) \right] \end{array} \right] \\ + \frac{N-I_g}{N} \left[\begin{array}{l} \sigma_{UN}^{I_g} \sqrt{\frac{2}{\pi}} e^{-\frac{[\mu_{UN}^{I_g}(\tilde{v}, \tilde{x})]^2}{2(\sigma_{UN}^{I_g})^2}} \\ + \mu_{UN}^{I_g}(\tilde{v}, \tilde{x}) \left[1 - 2\Phi \left(-\frac{\mu_{UN}^{I_g}(\tilde{v}, \tilde{x})}{\sigma_{UN}^{I_g}} \right) \right] \end{array} \right] \end{array} \right\} + \frac{1}{2} |\tilde{x}|. \quad (48) \end{aligned}$$

Then, given the distribution of (\tilde{v}, \tilde{x}) , its easy to compute $E(\tilde{q})$.

C. Proof of Equation (23).

The indirect utility of an informed trader in a group with I_g informed traders is

$$\begin{aligned} &VI(\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g}) \\ &= -\exp \left\{ \gamma c - \frac{[E(\tilde{v} | \tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g}) - \tilde{p}]^2}{2Var(\tilde{v} | \tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})} \right\}. \end{aligned}$$

His ex ante indirect utility is

$$VI^* = \sum_{I_g=0}^{N-1} \pi_{I_g}^{N-1} E [VI(\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})].$$

An argument similar to Grossman and Stiglitz (1980) shows that

$$\begin{aligned} & E [VI (\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})] \\ &= e^{\gamma c} \sqrt{\frac{Var (\tilde{v}|\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})}{Var (\tilde{v}|\tilde{p})}} E \left[-\exp \left(-\frac{[E (\tilde{v}|\tilde{p}) - \tilde{p}]^2}{2Var (\tilde{v}|\tilde{p})} \right) \right]. \end{aligned}$$

Thus,

$$\begin{aligned} VI^* &= e^{\gamma c} \sum_{I_g=0}^{N-1} \pi_{I_g}^{N-1} \sqrt{\frac{Var (\tilde{v}|\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})}{Var (\tilde{v}|\tilde{p})}} E \left[-\exp \left(-\frac{[E (\tilde{v}|\tilde{p}) - \tilde{p}]^2}{2Var (\tilde{v}|\tilde{p})} \right) \right] \\ &= \left[e^{\gamma c} \sum_{I_g=0}^{N-1} \pi_{I_g}^{N-1} \sqrt{\frac{\rho_v + \rho_\theta}{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right] E \left[-\exp \left(-\frac{[E (\tilde{v}|\tilde{p}) - \tilde{p}]^2}{2Var (\tilde{v}|\tilde{p})} \right) \right], \quad (49) \end{aligned}$$

where the second equality follows from the expressions of $Var (\tilde{v}|\tilde{p})$ (equation (12) with $I_g = 0$) and $Var (\tilde{v}|\tilde{p}; \tilde{s}_{i,g}, \tilde{y}_{1,g}, \dots, \tilde{y}_{I_g,g})$ (equation (9)).

Similarly, we can show that the ex ante indirect utility of an uninformed trader is

$$VU^* = \left[\sum_{I_g=0}^{N-1} \pi_{I_g}^{N-1} \sqrt{\frac{\rho_v + \rho_\theta}{\rho_v + \rho_\theta + I_g \rho_y}} \right] E \left[-\exp \left(-\frac{[E (\tilde{v}|\tilde{p}) - \tilde{p}]^2}{2Var (\tilde{v}|\tilde{p})} \right) \right]. \quad (50)$$

Therefore, by equations (49) and (50), $VI^* > VU^*$ if and only if

$$\frac{\sum_{I_g=0}^{N-1} \left(\pi_{I_g}^{N-1} \sqrt{\frac{\rho_v + \rho_\theta}{\rho_v + \rho_\theta + I_g \rho_y}} \right)}{\sum_{I_g=0}^{N-1} \left(\pi_{I_g}^{N-1} \sqrt{\frac{\rho_v + \rho_\theta}{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right)} > e^{\gamma c},$$

which gives the learning benefit function of (23).

D. Proof of Proposition 3.

The condition of $\sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + (\rho_x/\gamma^2)\rho_\varepsilon^2}} < e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$ is equivalent to $\hat{B}(1; 1) < 0 < \hat{B}(0; 1)$; that is, when $N = 1$, the endogenous fraction of informed μ^* takes interior values. This condition also ensures that for all values of $N > 1$, the value of μ^* is interior, because (i) the condition of $\hat{B}(0; N) = \hat{B}(0; 1) > 0$ ensures that $\mu^* > 0$, and (ii) the condition of $\hat{B}(1; N) < \hat{B}(1; 1) < 0$ ensures that $\mu^* < 1$.

As mentioned in the main text, the endogenous μ^* is determined by equation (27):

$$\hat{B}(\mu^*; N) = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \rho_\theta + (N-1)\mu^*\rho_y}} - e^{\gamma c} = 0,$$

which implies that

$$\rho_\theta + (N - 1) \rho_y \mu^* = \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right), \quad (51)$$

where $\left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right) > 0$ because $e^{\gamma c} < \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v}}$.

It is easy to show that $\frac{d\mu^*}{dN} < 0$. To see this, suppose $\frac{d\mu^*}{dN} \geq 0$. Then an increase in N will increase the left-hand side of equation (51), since $\frac{d\mu^*}{dN} \geq 0$ implies $\frac{d\rho_\theta}{dN} = \frac{d}{dN} \frac{\mu^{*2} [\rho_\varepsilon + (N-1)\rho_y]^2 \rho_x}{\gamma^2} \geq 0$ and $\frac{d(N-1)\rho_y \mu^*}{dN} \geq 0$. However, the right-hand side of equation (51) is independent of N . Thus, a contradiction.

Plugging the expression of $\rho_\theta = \frac{\mu^{*2} [\rho_\varepsilon + (N-1)\rho_y]^2 \rho_x}{\gamma^2}$ given in Proposition 1 into equation (51), we have

$$\frac{[\rho_\varepsilon + (N - 1) \rho_y]^2 \rho_x}{\gamma^2} \mu^{*2} + (N - 1) \rho_y \mu^* - \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right) = 0,$$

which is a quadratic function of μ^* . Solving μ^* yields

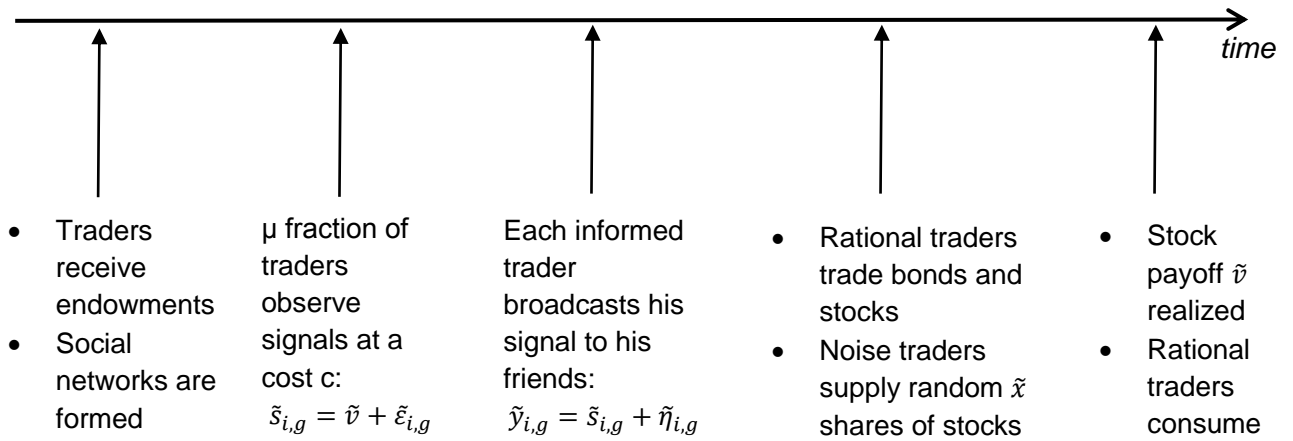
$$\mu^* = \frac{\frac{2}{(N-1)\rho_y} \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right)}{1 + \sqrt{1 + 4 \left(\frac{\rho_\varepsilon}{e^{2\gamma c} - 1} - \rho_v \right) \frac{\rho_x}{\gamma^2} \left(1 + \frac{\rho_\varepsilon}{(N-1)\rho_y} \right)^2}}.$$

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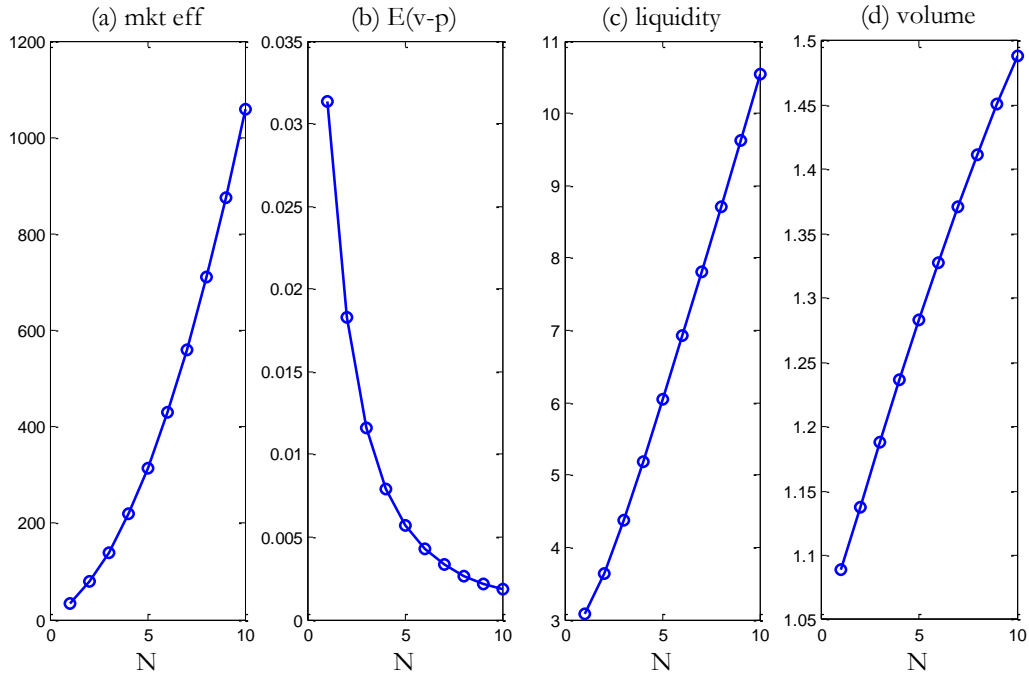
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Figure 1: Timeline



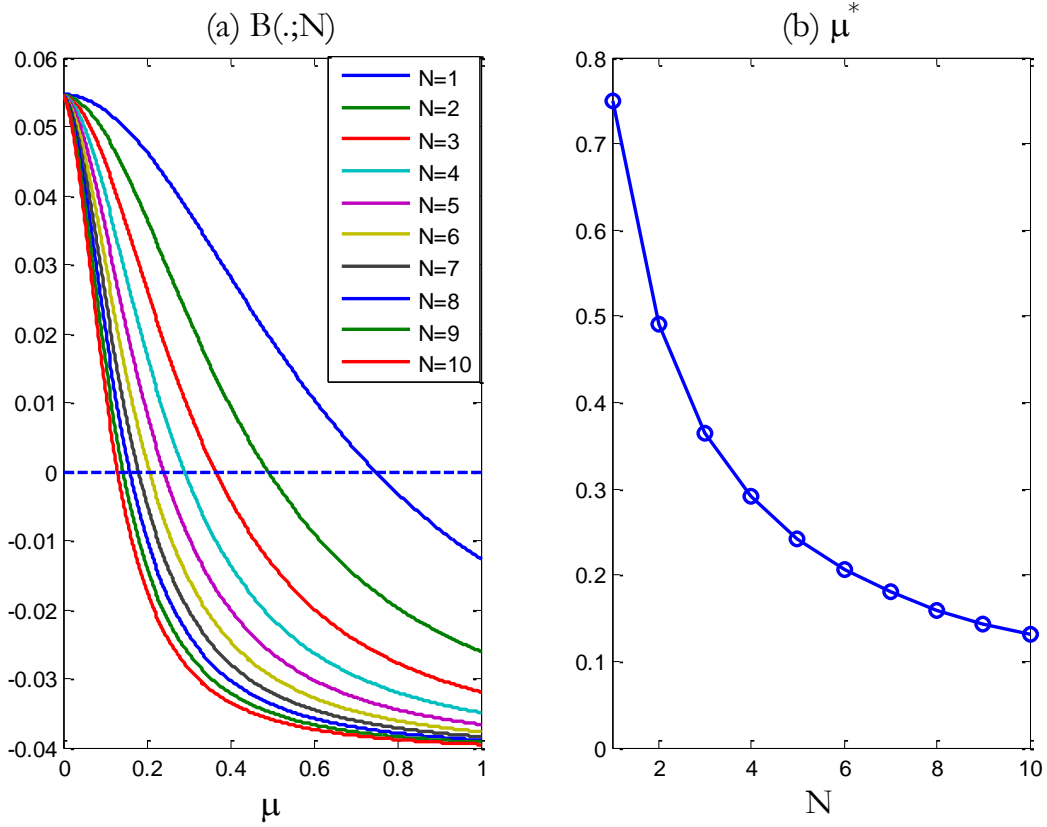
This figure plots the order of events.

Figure 2: Implications of Networks in Economies with Exogenous Information



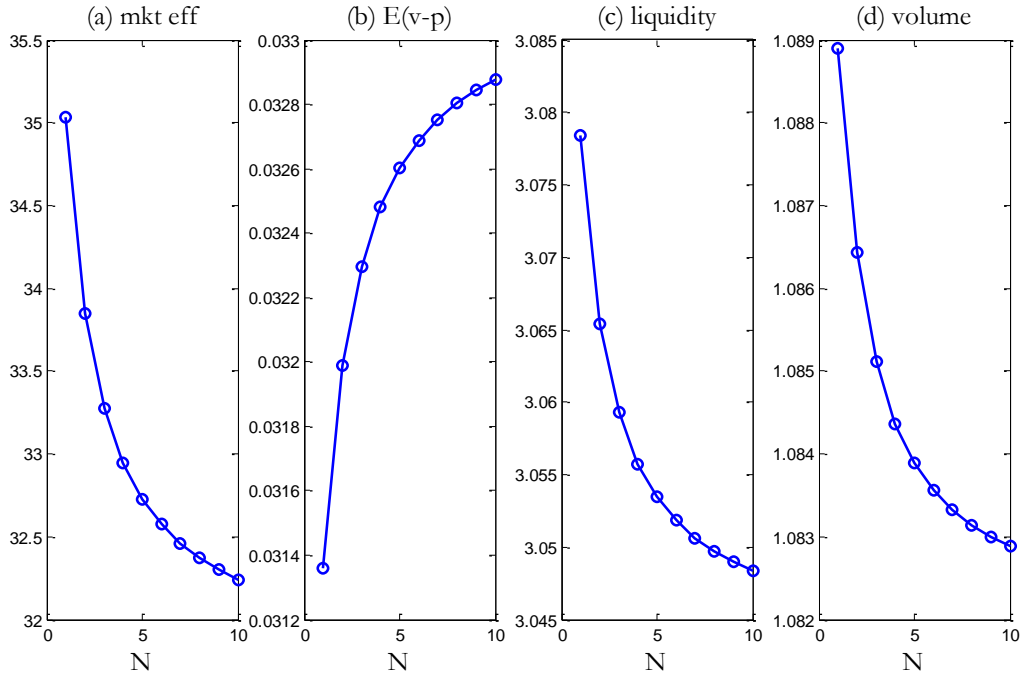
This figure plots how market efficiency (ρ_θ), cost of capital ($E(\tilde{v} - \tilde{p})$), liquidity ($1/\alpha_x$) and volume ($E(\tilde{q})$) vary with the connectedness N of networks in economies with exogenous information. The fraction of informed traders is set at $\mu=0.75$ for all economies. The other parameter values are: $\rho_v = 25$, $\rho_\varepsilon = \rho_\eta = 5$, $\rho_x = 10$, $\gamma=2$ and $\bar{v} = \bar{x} = 1$.

Figure 3: Learning Benefit and Information Market Equilibrium



This figure plots how the learning benefit function ($B(\cdot; N)$) and equilibrium fraction of informed traders (μ^*) vary with the connectedness N of networks. The parameter values are: $\rho_v = 25$, $\rho_\varepsilon = \rho_\eta = 5$, $\rho_x = 10$, $\gamma=2$, $\bar{v} = \bar{x} = 1$ and $c=0.02$.

Figure 4: Implications of Networks in Economies with Endogenous Information



This figure plots how market efficiency (ρ_{θ}^*), cost of capital ($E(\tilde{v} - \tilde{p})$), liquidity ($1/\alpha_x$) and volume ($E(\tilde{q})$) vary with the connectedness N of networks in economies with endogenous information. The parameter values are: $\rho_v = 25$, $\rho_\varepsilon = \rho_\eta = 5$, $\rho_x = 10$, $\gamma=2$, $\bar{v} = \bar{x} = 1$ and $c=0.02$.